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On the general fiber of an algebraic reduction of a compact complex manifold of algebraic codimension two

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1. Introduction

Let Z be a compact complex manifold of dimension three and of algebraic dimension one. In 1969 S. Kawai [4] has shown that a (bimeromorphically) ruled surface of genus $g \geq 2$ never appears as a general fiber of an algebraic reduction of Z. Ueno subsequently conjectured that the result will still be true in the higher dimensional case where Z is of dimension n and of algebraic dimension $n-2$ for any $n \geq 3$ (cf. [5, Remark 12.5]). The proof of Kawai of the above result depends on his Proposition 2 in [4], which can be stated as follows. Let $f : Z \to Y$ be a fiber space of compact complex manifolds with dim $Z = 3$ and dim $Y = 1$. (Here by a *fiber space* we mean a surjective holomorphic map with connected fibers.) Suppose that a general fiber F has the Hodge numbers $h^{2,0}(F) = 0$ and $h^{1,0}(F) > 0$. Then there exist a fiber space $h : S \to Y$ of curves over Y and a meromorphic map $\beta : Z \to S$ such that $f = h\beta$ and that for a smooth fiber $Z_y, y \in Y$, of f, the induced map $\beta_y : Z_y \to S_y$ is identified with the Albanese map onto its image. However, there seem counterexamples to this proposition in the case where F is bimeromorphically a ruled surface of genus one (cf. Section 3) and indeed the proof of that proposition in [4] seems insufficient even in the general case. In the present note we shall remark that by a slight modification of Kawai's proof, at least the statement at the beginning concerning ruled surfaces of genus ≥ 2 can be shown to hold true, and in fact even in a generalized form conjectured by Ueno. Note that another consequence of [4, Proposition 2] was also used by another authors [1, (3.5)].

2. Theorem

The precise statement is as follows.

Theorem 2.1. *Let $f : Z \to Y$ be a fiber space of compact complex manifolds which gives an algebraic reduction of Y. Then the general fiber F of f is never bimeromorphically equivalent to a ruled surface of genus ≥ 2.*

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