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A lower bound on the spectral gap of the 3-dimensional stochastic Ising models

By

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1. Introduction

Let us consider the Glauber dynamics at low temperature (large $\beta > 0$) which evolves on a cube $\Lambda_d(L) = (-L, L]^d \cap \mathbb{Z}^d$ ($L \in \mathbb{N}$) whose side-length is $2L$ with a boundary condition ω. By $\mathrm{gap}(\Lambda_d(L), \omega)$, we will denote the spectral gap corresponding to a boundary condition ω. Especially, By $\mathrm{gap}(\Lambda_d(L), \phi)$ and $\mathrm{gap}(\Lambda_d(L), +)$, we will mean spectral gaps corresponding to free and $+$ boundary conditions, respectively. L. E. Thomas proved in [Tho89] that

$$\mathrm{gap}(\Lambda_d(L), \phi) \le B \exp(-\beta C L^{d-1}) \qquad \text{for any} \quad L \in \mathbb{N} \tag{1.1}$$

for any $d \ge 2$ and sufficiently large $\beta > 0$, where $B = B(\beta, d) > 0$ and $C = C(\beta, d) > 0$. For $d = 2$ and any $\beta > \beta_c(2)$, it is known that the speed at which $\mathrm{gap}(\Lambda_2(L), +)$ shrinks to zero as $L \nearrow \infty$ is different from the one at which $\mathrm{gap}(\Lambda_2(L), \phi)$ does (see [Ma94], [Ma99] and [CGMS96]). In this paper, we confirm that it is also true for $d \ge 3$ and sufficiently large $\beta > 0$. In fact, we prove that for sufficiently large $\beta > 0$, some $B > 0$ and some $C > 0$,

$$\mathrm{gap}(\Lambda_d(L), +) \ge B \exp(-\beta C L^{d-2} (\log L)^2) \qquad \text{for any} \quad L \in \mathbb{N}. \tag{1.2}$$

For each $\delta \in [0, 1]$, we will consider the boundary condition η_δ which is defined by

$$\eta_\delta(x) = \begin{cases} +1 & \text{if} \quad x^d = -L \quad \text{and} \quad -\delta L < x^i \le \delta L \ (i \ne d), \\ 0 & \text{otherwise.} \end{cases} \tag{1.3}$$

For $d = 3$, we also prove that for sufficiently large $\beta > 0$, some $B > 0$ and some $C > 0$,

$$\mathrm{gap}(\Lambda_3(L), \eta_1) \ge B \exp(-\beta C L^{\frac{5}{3}} (\log L)^2) \qquad \text{for any} \quad L \in \mathbb{N}, \tag{1.4}$$

which implies at least that the speed at which $\mathrm{gap}(\Lambda_3(L), \eta_1)$ shrinks to zero as $L \nearrow \infty$ is different from the one at which $\mathrm{gap}(\Lambda_3(L), \phi)$ does, as was expected

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