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Erratum to "On the minimal solution for quasilinear degenerate elliptic equation and its blow-up" (J. Math. Kyoto Univ. Vol. 44 No. 2, 381–439)

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Proposition 8.2 in §8, namely the characterization of the behavior of solutions ψ_t as $t \to 0$ for a certain class of quasilinear elliptic equations, needs a correction about the support of their gradients. In the paper we used this property to have the uniform boundedness of $\psi_t, t \in [0,T]$ in th proof of Theorem 8.1, therefore it should be replaced by the next, in which the boundedness is simply given by a method of iteration.

Proposition 8.2. *Let $\varphi \in \tilde{V}_{\lambda,p}(\Omega)$ satisfy $|\nabla\varphi| = 0$ on $F_\varepsilon = \{x \in \Omega : dist(x, F_{\lambda,p}) \le \varepsilon\}$ for some $\varepsilon > 0$. Then there is a unique solution η_t of* (8.14) *for a small $T > 0$ such that $\eta_t = u_\lambda - t\psi_t$ for $\psi_t \in C^0([0,T], V_{\lambda,p}(\Omega))$ and*

$$\sup_{x\in\Omega, t\in[0,T]} |\psi_t| < \infty, \tag{8.1}$$

$$\lim_{t\to 0} ||\psi_t - \varphi||_{V_{\lambda,p}(\Omega\setminus F_\varepsilon)} = 0. \tag{8.2}$$

Proof. Since ∇u_λ does not vanish in $\overline{\Omega \setminus F_\varepsilon}$ and the nonlinearity $f \in C^1([0,\infty))$, first we see $u_\lambda \in C^{2,\sigma}(\overline{\Omega \setminus F_\varepsilon})$ for some $\sigma \in (0,1)$ as a solution to uniformly elliptic equation. By the theory of monotone operator $L_p(\cdot)$, there is a unique solution $\psi_t \in W_0^{1,p}(\Omega)$ for each t and $\nabla\psi_t$ is Hölder continuous function w.r.t. $x \in \Omega$. In §9, it is proved that $\psi_t - \varphi$ satisfies uniformly elliptic equation in $\Omega \setminus F_\varepsilon$ for a sufficiently small t. Hence by the elliptic regularity theory ψ_t can be assumed to be uniformly bounded in $C^2(\overline{\Omega \setminus F_\varepsilon})$ for a fixed small $\varepsilon > 0$. Since $L_p(\cdot)$ is differentiable in $W_0^{1,p}(\Omega)$, we have

$$\frac{L_p(u_\lambda - t\psi_t) - L_p(u_\lambda)}{t} = -\int_0^1 L_p'(u_\lambda - st\psi_t)\psi_t\, ds = -L_p'(u_\lambda)\varphi \in C^2(\overline{\Omega \setminus F_\varepsilon}).$$

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