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On the plurisubharmonicity of the leafwise Poincaré metric on projective manifolds

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Introduction

The aim of this paper is to give a new proof of the theorem asserting that the leafwise Poincaré metric has a plurisubharmonic variation. This theorem has been proved in [Br2], building on a method introduced in [Br1], and has later been generalised in [Br3] to the "relative" case. Below we shall recall the precise statement. The techniques used in these papers are sufficiently flexible to work on any compact Kähler manifold. Here, on the contrary, we shall restrict ourselves to complex projective manifolds, and the proof will follow a different strategy.

Let us resume the differences of the present paper with our previous ones.

Given a foliation by curves on a complex projective manifold X, we shall introduce below the *covering tube* U_S, which is roughly speaking a sort of global flow-box obtained by gluing together [Ily] the universal coverings of the leaves through a k-dimensional embedded disc $S \subset X$. It is a complex manifold fibered by curves over S and equipped with a meromorphic map into X. The matter is to prove that the fiberwise Poincaré metric on U_S has a plurisubharmonic variation. This is done in [Br1], [Br2] and [Br3] by showing that U_S has a suitable "holomorphic convexity" property, reducing in this way the problem to a clever result of [Yam].

Here, using the projectivity of X, we shall firstly remove from U_S an hypersurface, in such a way that the remaining part U_S'' appears as a Riemann domain over an Euclidean space. Then, using the Stein machinery, we shall prove that U_S'' has a special "pseudoconvex" exhaustion. This will allow to apply the main result of [M-Y] (more delicate than the corresponding particular case of [Yam] that we used before) to get that the fiberwise Bergman metric on U_S'' has a plurisubharmonic variation. Finally, by an L^2 removal–of–singularities argument, we shall pass from the Bergman metric on U_S'' to the Poincaré metric on U_S.

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