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On extensions of projective indecomposable modules

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Introduction

Let G be a finite group and p a prime. Let (K, R, k) be a p-modular system. We assume that K contains the $|G|$-th roots of unity and that k is algebraically closed. Suppose we are given a normal subgroup N of G such that G/N is a p-group and a G-invariant block b of N such that $N = QC_N(Q)$ for a defect group Q of b. Then, as is well-known, b has (up to isomorphism) a unique projective indecomposable RN-module V. It seems natural to ask whether there exists an extension U to G of V such that a vertex of U intersects N trivially. Let B be a unique block of G covering b. In Section 3, we obtain two necessary conditions such a module U must satisfy. Let P be a vertex of U and W a P-source of U. Then

(1) PQ is a defect group of B;

(2) W is an endo-permutation module, which is identified with a lift of a source of a unique simple kG-module in B.

(cf. Proposition 3.3, Corollary 3.17.)

In Section 4 we study the case where G/N is cyclic (and (1) holds for a p-subgroup P with $P \cap Q = 1$) and show that any indecomposable RG-module in B with vertex P and a P-source W as in (2) is actually an extension of V.

(Although we have mentioned only RG-modules, we also obtain similar results for kG-modules.)

In Section 1 we define an action of the group of capped endo-permutation modules over p-groups P (Dade [1, 2]) on the set of indecomposable P-modules. In Section 2 we determine vertices and sources of certain indecomposable modules.

Notation and convention

Let o denote R or k. For oG-modules V_i $(i = 1, 2)$, $V_1 \otimes V_2$ stands for $V_1 \otimes_o V_2$. Also for a direct product $G = G_1 \times G_2$ and oG_i-modules V_i $(i = 1, 2)$, $V_1 \times V_2$ stands for the external tensor product $V_1 \otimes_o V_2$. We denote by 1_G the trivial oG-module of rank one. For an RG-module U, let $U^* = U/\pi U$, where πR is the maximal ideal of R. For a kG-module X, an RG-module L such that

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