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The k-Buchsbaum property for some polynomial ideals

By

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Introduction

In order to define the topic of the title, we always assume that R is a standard graded ring over a field k and $\mathfrak{m}$ is the maximal homogeneous ideal. k-Buchsbaum graded modules M over R can be defined as having their local cohomology modules $H^i_{\mathfrak{m}}(M),\ 0 \leq i \leq d$, annihilated by $\mathfrak{m}^k$, where $d+1$ is the Krull-dimension of M. (For undefined terminology see [E].) They are natural generalizations of Cohen-Macaulay modules, which have $H^i_{\mathfrak{m}}(M) = 0,\ 0 \leq i \leq d$. A more workable definition for k-Buchsbaum ideals $\mathfrak{a} \subset K[x_0, \ldots, x_r] =: R_{r+1}$, where $\mathfrak{a}$ is a homogeneous ideal ($\delta(x_i) :=$ degree$(x_i) = 1,\ 0 \leq i \leq r$), is given below. An algorithm to test if such an ideal is perfect (i.e. $R_{r+1}/\mathfrak{a}$ is Cohen-Macaulay) or Buchsbaum (i.e. $R_{r+1}/\mathfrak{a}$ and $R_{r+1}/(\mathfrak{a}, F_0, \ldots, F_i),\ 0 \leq i \leq d$, are 1-Buchsbaum for any system of parameters (s.o.p.) $\{F_0, \ldots, F_d\}$) was given in [BV1] and [BV2]. Thus both of these papers deal with a fixed $k \in \{0, 1\}$ and do not address the question of an upper bound k, if $\mathfrak{a}$ is to be k-Buchsbaum for some k. The purpose of the present paper is to investigate this question without explicit computation of Ext-modules or local cohomology modules. We obtain an algorithm for certain binomial ideals. Although in [BH] it was shown, that no conclusive information about the k-Buchsbaum property of $\mathfrak{a}$ can be obtained from in$(\mathfrak{a})$ (the ideal of initial terms), our algorithm is based on the Gröbner bases calculations.

1. Homogeneous k-Buchsbaum ideals

We assume $R_{r+1} := K[x_0, \ldots, x_r],\ K$ an infinite field, $\mathfrak{a} \subset R_{r+1}$ a homogeneous ideal (with respect to the standard grading), $\dim(\mathfrak{a}) =$ Krull-dim$(R_{r+1}/\mathfrak{a})$ $= d+1$, without loss of generality $\{x_0, \ldots, x_d\}$ a s.o.p. for $\mathfrak{a}$ since K is infinite (i.e. the images $\{\bar{x_0}, \ldots, \bar{x_d}\}$ form a s.o.p. in $R_{r+1}/\mathfrak{a}$).

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