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3-graded decompositions of exceptional Lie algebras $\mathfrak{g}$ and group realizations of $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed}$ Part II, $G = E_7$, Cases 2, 3 and 4

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According to M. Hara [1], there are five cases of 3-graded decompositions $\mathfrak{g} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3$ of simple Lie algebras $\mathfrak{g}$ of type E_7. In the preceding paper [2], we gave the group realization of Lie subalgebras $\mathfrak{g}_{ev} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_2, \mathfrak{g}_0$ and $\mathfrak{g}_{ed} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_3$ of $\mathfrak{g}$ of Case 1. In the present paper, we give the group realization of $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed}$ of Cases 2, 3 and 4. We rewrite the results of $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed}$ of Cases 2, 3 and 4.

Case 2	$\mathfrak{g}$	$\mathfrak{g}_{ev}$ $\mathfrak{g}_{ed}$	$\mathfrak{g}_0$ $\dim \mathfrak{g}_1, \dim \mathfrak{g}_2, \dim \mathfrak{g}_3$
	${\mathfrak{e}_7}^C$	$\mathfrak{sl}(2, C) \oplus \mathfrak{so}(12, C)$ $C \oplus \mathfrak{sl}(7, C)$	$C \oplus C \oplus \mathfrak{sl}(6, C)$ $26, 16, 6$
	$\mathfrak{e}_{7(7)}$	$\mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{so}(6, 6)$ $\boldsymbol{R} \oplus \mathfrak{sl}(7, \boldsymbol{R})$	$\boldsymbol{R} \oplus \boldsymbol{R} \oplus \mathfrak{sl}(6, \boldsymbol{R})$ $26, 16, 6$

Case 3	$\mathfrak{g}$	$\mathfrak{g}_{ev}$ $\mathfrak{g}_{ed}$	$\mathfrak{g}_0$ $\dim \mathfrak{g}_1, \dim \mathfrak{g}_2, \dim \mathfrak{g}_3$
	${\mathfrak{e}_7}^C$	$C \oplus {\mathfrak{e}_6}^C$ $C \oplus \mathfrak{so}(12, C)$	$C \oplus C \oplus \mathfrak{so}(10, C)$ $17, 16, 10$
	$\mathfrak{e}_{7(7)}$	$\boldsymbol{R} \oplus \mathfrak{e}_{6(6)}$ $\boldsymbol{R} \oplus \mathfrak{so}(6, 6)$	$\boldsymbol{R} \oplus \boldsymbol{R} \oplus \mathfrak{so}(5, 5)$ $17, 16, 10$
	$\mathfrak{e}_{7(-25)}$	$\boldsymbol{R} \oplus \mathfrak{e}_{6(-26)}$ $\boldsymbol{R} \oplus \mathfrak{so}(2, 10)$	$\boldsymbol{R} \oplus \boldsymbol{R} \oplus \mathfrak{so}(1, 9)$ $17, 16, 10$

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