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Fujita's approximation theorem in positive characteristics

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Introduction

Let X be a projective variety of dimension n over an algebraically closed field k.

For any line bundle L on X, we define the *volume* of L to be:

$$\mathrm{vol}_X(L) := \limsup_{m\to\infty} \frac{h^0(X, L^{\otimes m})}{m^n/n!}$$

It is known that this function vol_X can be extended to a homogeneous, continuous real valued function on $\mathrm{N}^1(X)_{\mathbb{R}}$.

The main theorem of this paper is as follows:

Theorem 0.1 (Fujita's approximation theorem)**.** *Let $\xi \in \mathrm{N}^1(X)_{\mathbb{Q}}$ be a rational big class. Then, for an arbitrary small real number $\epsilon > 0$, there exist a birational morphism $\pi : X' \to X$ of projective varieties and a decomposition*

$$\pi^*\xi = \alpha + e$$

in $\mathrm{N}^1(X')_{\mathbb{Q}}$, which satisfy the following conditions:

(i) *α is an ample class and e is effective.*
(ii) $\mathrm{vol}_{X'}(\alpha) > \mathrm{vol}_X(\xi) - \epsilon$.

Note that the characteristic of the base field k could be positive. The proof of Fujita's approximation theorem in the original paper [3] uses Hironaka's desingularization theorem. Other proofs, obtained by Lazardsfeld [11] and Nakamaye [13], also uses the desingularization theorem. In this sense, Fujita's approximation theorem can be applied only to the case where characteristic is zero. In this paper, we verified this theorem in positive characteristics. The idea is to use de Jong's alteration theorem (see Theorem 2.1) as a substitute for Hironaka's theorem.

This paper consists of 3 sections.

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