J. Math. Kyoto Univ. (JMKYAZ)
48-2 (2008), 401–443

Effective calculation of the geometric height and the Bogomolov conjecture for hyperelliptic curves over function fields

By

Kazuhiko Yamaki

Introduction

Let us begin with a brief survey on the Bogomolov conjecture. Let A be an abelian variety over a field K. We assume that K is a number field for a while. Let L be a symmetric ample line bundle on A and $\hat{h}_L$ the Néron-Tate height on $A(\overline{K})$, where $\overline{K}$ is the algebraic closure of K. For a closed subvariety V of $A \otimes_K \overline{K}$, we set

$$V(\epsilon) := \{P \in V(\overline{K}) \mid \hat{h}_L(P) \leq \epsilon\}.$$

Theorem ([14]. Generalized Bogomolov conjecture). *Suppose that V is not the translation of an abelian subvariety by a torsion point. Then there exists $\epsilon > 0$ such that $V(\epsilon)$ is not Zariski dense in V.*

This theorem was proved by Zhang in [14]. The original version due to Bogomolov deals with a curve V embedded in its Jacobian variety, which was proved by Ullmo in [8].

Let us recall the proof of Zhang. In [13], he introduced the notion of admissible metric on a line bundle on V, and defined the admissible intersection numbers and the admissible height, which are compatible with the Néron-Tate height. Then, he found a key inequality called the fundamental inequality:

$$\sup_{W \subsetneq V} \left\{ \inf_{x \in (V \setminus W)(\overline{K})} \hat{h}_L(x) \right\} \geq \text{“the admissible height of } V\text{”},$$

where W ranges over all proper closed subvarieties of V. If the admissible height is proved to be positive, the fundamental inequality leads the Bogomolov conjecture immediately, but in general it is quite hard to calculate. To avoid this difficulty, Zhang proved the equidistribution theorem which says that a certain kind of sequence of small points should be equidistributed in the complex

Received January 7, 2008
Revised February 19, 2008