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p-compact groups as subgroups of maximal rank of Kac-Moody groups

By

Jaume Aguadé*

1. Introduction

In [28], Kitchloo constructed a map $f : BX \to BK^\wedge_p$ where K is a certain Kac-Moody group of rank two, X is a rank two mod p finite loop space and f is such that it induces an isomorphism between even dimensional mod p cohomology groups. Here B denotes the classifying space functor and $(-)^\wedge_p$ denotes the Bousfield-Kan $\mathbb{F}_p$-completion functor ([8]).

This space X —or rather the triple $(X^\wedge_p, BX^\wedge_p, e)$ where $e : X \simeq \Omega BX$— is a particular example of what is known as a p-compact group. These objects were introduced by Dwyer and Wilkerson in [15] as the homotopy theoretical framework to study finite loop spaces and compact Lie groups from a homotopy point of view. The foundational paper [15] together with its many sequels by Dwyer-Wilkerson and other authors represent now an active, well established research area which contains some of the most important recent advances in homotopy theory.

While p-compact groups are nowadays reasonably well understood objects, our understanding of Kac-Moody groups and their classifying spaces from a homotopy point of view is far from satisfactory. The work of Kitchloo in [28] started a project which has also involved Broto, Saumell, Ruiz and the present author and has produced a series of results ([2], [3], [10]) which show interesting similarities between this theory and the theory of p-compact groups, as well as non trivial challenging differences.

The goal of this paper is to extend the construction of Kitchloo that we have recalled above to produce rank-preserving maps $BX \to BK^\wedge_p$ for a wide family of p-compact groups X. These maps can be understood as the homotopy analogues to monomorphisms, in a sense that will be made precise in Section 13. We prove:

Theorem 1.1. *Let p be a prime and let X be a simply connected p-compact group with Weyl group W_X. Assume that the order of W_X is prime to*

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