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QUANTUM BOHR COMPACTIFICATION

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ABSTRACT. We introduce a non commutative analog of the Bohr compactification. Starting from a general quantum group G we define a compact quantum group $\mathfrak{b}\mathsf{G}$, which has a universal property such as the universal property of the classical Bohr compactification for topological groups. We study the object $\mathfrak{b}\mathsf{G}$ in the special cases when G is a classical locally compact group, the dual of a classical group, a discrete or compact quantum group, and a quantum group arising from a manageable multiplicative unitary. We also use our construction to give new examples of compact quantum groups.

1. Introduction

Let A be a C^*-algebra and let $\Delta \in \mathrm{Mor}(A, A \otimes A)$ be a coassociative morphism. Then $\mathsf{G} = (A, \Delta)$ is a non commutative analog of a semigroup. If A is unital and the sets

$$\begin{aligned}&\left\{\Delta(a)(I \otimes b) :\ a, b \in A\right\},\\ &\left\{(a \otimes I)\Delta(b) :\ a, b \in A\right\}\end{aligned} \tag{1.1}$$

are linearly dense in $A \otimes A$, then G is a *compact quantum group* ([33]). In the papers [29], [30], [33] S.L. Woronowicz brought the understanding of compact quantum groups to a very satisfactory level. His theory is a cornerstone of the theory of locally compact quantum groups, the latter being still at the development stage. There are different approaches to defining general quantum groups. The common agreement is that a quantum group is described by a pair (A, Δ) of a C^*-algebra and a comultiplication on A and possessing some additional properties. The linear density (and containment) of the sets (1.1) in $A \otimes A$ is generally accepted. A very successful definition of a reduced C^*-algebraic quantum group is due to Kustermans and Vaes ([9]). A related notion of an algebraic quantum group was introduced by van Daele (see, e.g.,

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