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PROBABILISTIC INVARIANT MEASURES FOR NON-ENTIRE FUNCTIONS WITH ASYMPTOTIC VALUES MAPPED ONTO ∞

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Abstract. We study the dynamics of transcendental meromorphic functions of the form $f(z) = R \circ \exp(z)$, where R is a non-constant rational map and both asymptotic values $R(0)$ and $R(\infty)$ are eventually mapped onto ∞. With each map f we associate its projection F on the cylinder $\mathcal{P}$. Let J_F^r consist of all points whose trajectory returns infinitely often to some compact set whose intersection with the postsingular set is empty, and let $h = \mathrm{HD}(J_F^r)$ be the Hausdorff dimension of J_F^r. We prove that the h-dimensional Hausdorff measure H^h of J_F^r is positive and finite, while the h-dimensional packing measure of J_F^r is locally infinite at every point of this set. We also prove that there exists a unique F-invariant Borel probability measure μ on J_F^r that is absolutely continuous with respect to the Hausdorff measure H^h, and that μ is ergodic and conservative.

1. Introduction

We consider the family $\mathcal{R}$ of transcendental meromorphic functions $f(z) : \mathbb{C} \to \overline{\mathbb{C}}$ of the form

$$f(z) = R \circ \exp(z), \tag{1.1}$$

where R is a non-constant rational map. The set of singularities $\mathrm{Sing}(f^{-1})$ consists of finitely many critical values and two asymptotic values

$$\xi_1 := R(0), \quad \xi_2 := R(\infty).$$

Let $\mathcal{Q}^*$ be the class of non-entire functions from $\mathcal{R}$ such that both asymptotic values are mapped onto infinity, i.e., there exist numbers $q_i > 1$, $i = 1, 2$, such

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