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ON THE ZEROS OF POLYNOMIALS WITH RESTRICTED COEFFICIENTS

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1. Introduction

The study of the location of zeros of polynomials from

$$\mathcal{F}_n := \left\{ p\colon\ p(x) = \sum_{i=0}^{n} a_i x^i\,,\quad a_i \in \{-1, 0, 1\} \right\}$$

begins with Bloch and Pólya [2]. They prove that the average number of real zeros of a polynomial from $\mathcal{F}_n$ is at most $c\sqrt{n}$. They also prove that a polynomial from $\mathcal{F}_n$ cannot have more than

$$\frac{cn \log\log n}{\log n}$$

real zeros. This result, which appears to be the first on this subject, shows that polynomials from $\mathcal{F}_n$ do not behave like unrestricted polynomials. Schur [11] and by different methods Szegő [12] and Erdős and Turán [7] improve the above bound to $c\sqrt{n \log n}$ (see also [5]).

In [6] we give the right upper bound of $c\sqrt{n}$ for the number of real zeros of polynomials from a large class, namely for all polynomials of the form

$$p(x) = \sum_{j=0}^{n} a_j x^j, \qquad |a_j| \le 1\,, |a_0| = |a_n| = 1\,, a_j \in \mathbb{C}\,.$$

In this paper we extend this result by proving that a polynomial of the form

$$p(x) = \sum_{j=0}^{n} a_j x^j, \qquad |a_j| \le 1\,, |a_0| = 1\,, a_j \in \mathbb{C}\,,$$

cannot have more than $c\sqrt{n}$ zeros inside any polygon with vertices on the unit circle, where c depends only on the polygon.

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