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A GENERALIZED LITTLEWOOD THEOREM FOR WEINSTEIN POTENTIALS ON A HALFSPACE

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1. Introduction and statement of results

Let $\mathbb{R}^n_+ = \{x = (x_1, \ldots, x_n) \in \mathbb{R}^n \colon x_n > 0\}$ denote the upper halfspace in $\mathbb{R}^n$, $n \geq 2$. We view the boundary of $\mathbb{R}^n_+$ as $\mathbb{R}^{n-1}$. Let $k \in \mathbb{R}$. The Weinstein equation with parameter k is $L_k(f) = 0$ where

$$L_k(f) = \sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_j^2} + \frac{k}{x_n}\frac{\partial f}{\partial x_n}.$$

The C^2 functions which satisfy the Weinstein equation form a Brelot harmonic space [He]. We shall refer to these solutions as L_k-harmonic functions. The L_0-harmonic functions are just the classical harmonic functions. An integral representation for all positive L_k-harmonic functions in terms of measures on $\mathbb{R}^{n-1} \cup \{\infty\}$ (when we simply use the term measure, we mean a nonnegative, regular, Borel measure) was given in [BCB1]. There, the uniqueness of such an integral was demonstrated using Choquet's theorem. The same authors have also proved that every positive L_k-harmonic function has finite non-tangential limit at (Lebesgue) almost every point in $\mathbb{R}^{n-1}$ [BCB2].

In our paper we consider the boundary behavior of L_k-potentials. We recall that L_k-superharmonic functions, following the axiomatic study in [He], are precisely those lower semicontinuous, $(-\infty, \infty]$ valued functions v that satisfy $L_k(v) \leq 0$ in the sense of distributions. The L_k-potentials (the Weinstein potentials of the title) are those positive L_k-superharmonic functions that majorise no positive L_k-harmonic function. For every $y \in \mathbb{R}^n_+$ we associate the function

$$G_k(x, y) = a_{n,k} x_n^{1-k} y_n \int_0^{\pi} \frac{\sin^{1-k} t}{[|x-y|^2 + 2x_n y_n(1-\cos t)]^{(n-k)/2}}\, dt \quad \text{for } k \leq 1, \tag{1}$$

and

$$G_k(x, y) = a_{n,2-k} y_n^{k} \int_0^{\pi} \frac{\sin^{k-1} t}{[|x-y|^2 + 2x_n y_n(1-\cos t)]^{(n+k-2)/2}}\, dt \quad \text{for } k \geq 1, \tag{2}$$

where

$$a_{n,k} = \frac{\Gamma\left(\frac{n-k}{2}\right)}{2\pi^{n/2}\Gamma\left(\frac{2-k}{2}\right)} \quad \text{for } k \leq 1.$$

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