ILLINOIS JOURNAL OF MATHEMATICS
Volume 41, Number 4, Winter 1997

VECTOR-VALUED ANALYTIC FUNCTIONS OF BOUNDED MEAN OSCILLATION AND GEOMETRY OF BANACH SPACES

OSCAR BLASCO

Introduction

When dealing with vector-valued functions, sometimes is rather difficult to give non trivial examples, meaning examples which do not come from tensoring scalar-valued functions and vectors in the Banach space, belonging to certain classes. This is the situation for vector valued *BMO*. One of the objectives of this paper is to look for methods to produce such examples.

Our main tool will be the vector-valued extension of the following result on multipliers, proved in [MP], which says that the space of multipliers between H^1 and *BMOA* can be identified with the space of Bloch functions $\mathcal{B}$, i.e. $(H^1, BMOA) = \mathcal{B}$ (see Section 3 for notation), which, in particular gives $g * f \in BMOA$ whenever $f \in H^1$ and $g \in \mathcal{B}$.

Given two Banach spaces X, Y it is rather natural to define the convolution of an analytic function with values in the space of operators $\mathcal{L}(X, Y)$, say $F(z) = \sum_{n=0}^{\infty} T_n z^n$, and a function with values in X, say $f(z) = \sum_{n=0}^{\infty} x_n z^n$, as the function given by $F * g(z) = \sum_{n=0}^{\infty} T_n(x_n) z^n$.

It is not difficult to see that the natural extension of the multipliers' result to the vector-valued setting does not hold for general Banach spaces. To be able to get a proof of such a result we shall be using the analogue of certain inequalities, due to Hardy and Littlewood [HL3], in the vector valued setting, namely

$$\left(\int_0^1 (1-r) M_1^2(f', r)dr\right)^{\frac{1}{2}} \leq C||f||_{H^1}$$

and its dual formulation

$$||f||_* \leq C \left(\int_0^1 (1-r) M_\infty^2(f', r)dr\right)^{\frac{1}{2}}.$$

This leads us to consider spaces where these inequalities hold when the absolute value is replaced by the norm in the Banach space, which turn out to be very closely related to notions as (Rademacher) cotype 2 and type 2.

Received December 29, 1995.
1991 Mathematics Subject Classification. Primary 46B20 , 46E40.
The author was partially supported by the Spanish DGICYT, Proyectos PB92-0699 and PB95-0261.