ON THE NUMBER OF CERTAIN TYPES OF POLYHEDRA

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1. Introduction. The discussions in the following pages are concerned with certain enumerations in the morphology of Eulerian polyhedra in 3-space. The "morphology" is actually the topology of the complexes consisting of the vertices, edges, and faces of polyhedra, with the restriction that these elements are linear. A class of polyhedra isomorphic to each other with respect to incidences is called a *type*. We impose also on this isomorphism the condition that it preserves the orientation. We shall call such a class a "type in the strict sense" in contradistinction to the usage in most classical papers in which the preservation of orientation was not required (Kirkman, Brückner). In Brückner's book [1] there is to be found the explicit statement "Verschiedene Typen ergeben sich nur, wenn . . ., denn weiterhin treten die Spiegelbilder der bisherigen Vielflache auf". That means that mirror-symmetric polyhedra belong in this sorting to the same type (in our terminology employed here "type in the wider sense"). Steinitz [9] in most parts of his book shares Brückner's point of view. However, on p. 86 he speaks of "direct isomorphy", which he defines as isomorphy under preservation of orientation. He formulates there his famous theorem on convex polyhedra, which is actually a homotopy theorem, stating that two convex polyhedra of the same type in the strict sense are homotopically equivalent, again with the "morphological" restriction that the complexes which constitute the continuous transition from one convex polyhedron to a directly isomorphic one remain always convex polyhedra of the same type in the strict sense.

It is clear that the number of types in the strict sense is at least as great as the number of types in the wider sense.

The number of types of polyhedra of a given number F of faces, whether the types are counted in the wider or the strict sense, is a problem mentioned by Euler, Steiner, Kirkman [4], Eberhardt [3], Brückner [1], and Steinitz [8]. Usually attention is only paid to trihedral polyhedra, i.e. those whose every vertex belongs to 3 faces and 3 edges. These polyhedra are considered as "general", whereas those with vertices of higher incidence are looked upon in such discussions as degenerate. We shall in this article also deal only with *trihedral polyhedra* and shall no longer mention this restriction.

For the types in the wider sense the enumeration has been carried out up to $F = 11$ by Brückner and recently, with the help of an electronic computer, by D. W. Grace, a student of G. Pólya. The number $\psi(F)$ of types (in the wider sense) increases rapidly with F, and no general formula has been found for it.

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