A CHARACTERIZATION OF CERTAIN REGULAR d-CLASSES IN SEMIGROUPS

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The concept of a d-class in a semigroup was introduced and investigated by Green [3]. The importance of this concept in the study of semigroups is indicated in [2]. Regular d-classes have been studied by Miller and Clifford [2], [4]. A semigroup consisting of a single d-class is called bisimple. Clifford [1] has determined the structure of all bisimple inverse semigroups with identity.

Let S be a semigroup and A a non-empty subset of S. Let E_A denote the collection of idempotents of A. E_A may be partially ordered as follows: $e \leq f$ if and only if $ef = fe = e$. We characterize regular d-classes D for which E_D is linearly ordered and we determine the structure of bisimple inverse semigroups G for which E_G is linearly ordered. The connection between certain regularity conditions [2] and the linear ordering of idempotents is considered.

Two elements of S are said to be R-(L-) equivalent if they generate the same principal right (left) ideal. Two elements a, b of S are d-equivalent if there exists x in S such that $a\ R\ x$ and $x\ L\ b$ (or equivalently there exists y in S such that $a\ L\ y$ and $y\ R\ b$). An element a in S is called right (left) regular if $a\ R\ a^2$ ($a\ L\ a^2$). a is called biregular if it is either right regular or left regular. S is said to be biregular if all its elements are biregular. An element a in S is regular if a in aSa. A subset of S is regular if all its elements are regular. A regular semigroup in which the idempotents commute is called an inverse semigroup [2], [5].

Let e be an idempotent element of S. $P_e(Q_e)$ will denote the right (left) unit subsemigroup of eSe (the set of elements of eSe having a right (left) inverse with respect to e the identity of eSe). H_e will denote the group of units of eSe.

By a decomposition of S we mean a partition of S into a union of disjoint subsemigroups.

S^1 will denote S with an appended identity [2, p. 4].

LEMMA. *Let S be a bisimple inverse semigroup. Then E_S is linearly ordered if and only if S is biregular.*

Proof. Suppose that E_S is linearly ordered. Then, if a in S there exist e, f in E_D such that $a\ R\ e$ and $a\ L\ f$ [2], [4]. If $ef = fe = e$, then $aea\ L\ fea$ or $a^2\ L\ a$ [2], [4]. If $ef = fe = f$, $a^2\ R\ a$. Conversely, suppose that S is biregular. If e, f in E_D , there exists a in D such that $e\ R\ a$ and $a\ L\ f$. Hence, if $a\ R\ a^2$ then $a\ R\ ae$. Since $a\ L\ f$, there exists x in S such that $xa = f$. Thus, $xa\ R\ xae$

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