CONVERGENCE ON FILTERS AND SIMPLE EQUICONTINUITY[1]

BY

JOHN W. BRACE

Convergence on filters is conceived as a replacement for simple uniform convergence [5, Dictionnaire]. Besides retaining the notable characteristic of simple uniform convergence, preservation of continuity, the new convergence is intimately related to the linear topological structure of function spaces.

The first section defines the concept of convergence on a filter and shows that it leads to a necessary and sufficient condition for a filter of functions continuous at a point x to converge at x to a function which is also continuous there. The second section develops the associated uniformity and shows that the topology of almost uniform convergence is the special case of convergence on all ultra filters [2], [3]. The last of the four applications given in Section three can be considered as a localization of the method used in obtaining a Stone-Čech compactification.

Section four presents a weakened form of equicontinuity called simple equicontinuity. The interesting properties which it has in common with equicontinuity are displayed. It is used to characterize the relatively compact sets for the topology of pointwise convergence in the space of continuous functions. The result is an analogue of Ascoli's theorem [9]. Combining the present result with Ascoli's theorem leads to another characterization of compact sets.

The last section examines simple equicontinuity in a locally convex linear topological space. It terminates in a strengthened form of the Alaoglu-Bourbaki theorem in which simple equicontinuity replaces equicontinuity [6], [8].

1. Convergence on a filter

Throughout this paper $G(S, E)$ denotes a space of functions whose common domain is a set S and whose ranges are in the Hausdorff uniform space E. The space E^S of all functions from S into E is denoted by $J(S, E)$.

The concept of simple uniform convergence originated with Dini [7]. To observe its relationship to convergence on a filter, consider the filter $\mathfrak{F}$ of Definition 1.2 as the filter of neighborhoods of the point s_0 in Definition 1.1. Besides the obvious replacement of the sequence $\{f_n\}$ by the filter $\mathcal{G}$, note that no pointwise convergence is required in 1.2 and every refinement of the filter $\mathcal{G}$ will also converge on $\mathfrak{F}$.

1.1 DEFINITION. (*Simple Uniform Convergence*) [5, Dictionnaire]. A se-

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