# PROJECTIVE REPRESENTATIONS OF FINITE GROUPS IN CYCLOTOMIC FIELDS

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## Introduction

In [2] Brauer proved that every representation of a finite group $G$ in the field $C$ of complex numbers is equivalent in $C$ to a representation of $G$ in the field of the $|G|$-th roots of unity, and in [3] he improved this by replacing $|G|$ by the exponent of $G$. In this paper we consider the corresponding question for projective representations. Our main result is contained in the following theorem.

THEOREM. *Every projective representation $\mathfrak{X}$ of $G$ in $C$ is projectively equivalent (see Section 2) in $C$ to a projective representation $\mathfrak{Z}$ of $G$ in the field of the $|G|$-th roots of unity. $\mathfrak{Z}$ can be chosen so that its factor set takes on only $|G|$-th roots of unity as values, and so that it is inflated from any quotient group $G/H$ from which the factor set of $\mathfrak{X}$ is inflated.*

This result is given in a more precise form in Theorems 5 and 6, which also include a similar result for modular projective representations (see [10], [11]). It would be of interest to know whether $|G|$ could be replaced by the exponent of $G$ in these results.

Our method combines those of Brauer [3] and Schur [13]. In Section 1 we give a modification (Theorem 1) of the Brauer induction theorem [4], [6, p. 283] which takes into account the behavior of characters on a given subgroup of the center $Z(G)$ of $G$; and we use this to prove in Theorem 3 that every representation of every subgroup of $G$ in $C$ is equivalent to a representation in the field of the $|G : Z(G) \cap G'|$-th roots of unity, where $G'$ is the commutator subgroup of $G$. Schur's method is then applied in Section 2 to obtain the main result. In the final section we show that some basic results of Clifford [5] and Mackey [9] can be obtained within the field of the $|G|$-th roots of unity.

## 1. Characters and representations

Our first result is a modification of the Brauer induction theorem.

THEOREM 1. *Let $A$ be a subgroup of the center of a finite group $G$; let $\omega$ be a linear character of $A$. Then every irreducible character $\chi$ of $G$ such that $\chi \mid A$ contains $\omega$ can be expressed in the form*

$$\chi = \sum_i c_i \lambda_i^G, \tag{1}$$

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