A PROPERTY OF A CLASS OF DISTRIBUTIONS ASSOCIATED WITH THE MINKOWSKI METRIC

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It is a well-known fact that if a sufficiently differentiable function f on $R^n = \{\langle t_1, \cdots, t_n\rangle : t_1, \cdots, t_n \text{ real}\}$, $n \geqq 2$, satisfies the wave equation

$$\Box f = \partial^2 f/\partial t_1^2 - \partial^2 f/\partial t_2^2 - \cdots - \partial^2 f/\partial t_n^2 = 0$$

and $f = \partial f/\partial t_1 = 0$ on the *disk* $t_1 = a_1$, $(t_2 - a_2)^2 + \cdots + (t_n - a_n)^2 \leqq \beta^2$, when $a_1, \cdots, a_n$ are real and $\beta > 0$, then $f = 0$ throughout the double conical region

$$| t_1 - a_1 | + [(t_2 - a_2)^2 + \cdots + (t_n - a_n)^2]^{1/2} \leqq \beta.$$

The same conclusion holds if $P(\Box)f = 0$ where P is a polynomial of degree k with real roots and $f = \partial f/\partial t_1 = \cdots = \partial^{2k-1}f/\partial t_1^{2k-1} = 0$ on the disk.

The solutions of $P(\Box)f = 0$ which are tempered distributions can be characterized as the Fourier transforms of tempered distributions concentrated in the finitely many hyperboloids $x_1^2 - x_2^2 - \cdots - x_n^2 = (\text{root of } P)/4\pi^2$, which may involve derivatives perpendicular to a hyperboloid only to a degree up to one less than the multiplicity of the corresponding root of P.

The object of this paper is to prove that if a tempered distribution $T = T(x_1, \cdots, x_n)$ is, in a suitable sense, of faster than exponential decrease as $| x_1^2 - x_2^2 - \cdots - x_n^2 |^{1/2} \to \infty$, its Fourier transform is determined throughout each double conical region as described above by its values arbitrarily near the corresponding disk. A somewhat misformulated version of this result appeared in my doctoral dissertation at Princeton University, written while on a National Science Foundation Cooperative Fellowship (1961–62). Thanks are due to Professors G. A. Hunt and Edward Nelson for reading several earlier drafts and making helpful comments.

For any n-tuple $z = \langle z_1, \cdots, z_n\rangle$ of complex numbers, $n \geqq 2$, we will let

$$_2| z |_n^2 = z_2^2 + \cdots + z_n^2, \quad \text{and} \quad \| z \|^2 = z_1^2 - {}_2| z |_n^2.$$

Let $Q(R^n)$, $n \geqq 2$, be the space of C^∞ complex-valued functions f on R^n such that for some $\beta > 0$, there is for every $m > 0$ a $K > 0$ such that

$$| f(x) | = | f(x_1, \cdots, x_n) | \leqq K \exp(\beta \,|\, \| x \|^2 |^{1/2})/(1 + x_1^2 + \cdots + x_n^2)^m$$

for all $x_1, \cdots, x_n$, with every partial derivative of f, of any order, satisfying the same conditions, possibly with different values of K. We define a pseudotopology in Q as follows: $f_k \to 0$ in Q if and only if β and K can be chosen independently of k (the latter for each partial derivative and $m > 0$), and

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