THE SINGULARITIES, S_1^q

BY

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Introduction

In this paper all manifolds and maps are either real C^∞ or complex analytic. A submanifold is always a regularly embedded submanifold, that is, the inclusion map into the ambient manifold is a homeomorphism into (real C^∞ or complex analytic).

Let V and M be manifolds of dimensions n and p respectively, and let $s = \min(n, p)$. If f is a map of V in M, let $S_1(f)$ be the set of all $v \in V$ such that rank $f_* = s - 1$ at v; here f_* means the induced map on tangent spaces. If $S_1(f)$ is a submanifold of V, we define $S_1^2(f)$ to be $S_1(f \mid S_1(f))$. In this way, for "sufficiently nice" maps, we may proceed letting $S_1^q(f) = S_1(f \mid S_1^{q-1}(f))$. This is the definition of Thom [7].

In Theorem 1, S_1^q are described "universally" independent of the map. That is, S_1^q are submanifolds of J^q, the space of q-jets at the origin of maps of n-space in p-space, such that if f maps V in M and the induced jet mapping $J^q(f) : V \to J^q(V, M)$ is transversal to all the $S_1^q(V, M)$, then

$$S_1^q(f) = (J^q(f))^{-1}(S_1^q(V, M)).$$

Here $J^q(V, M)$ is the bundle over $V \times M$ with fibre J^q and group the group of q-jets of coordinate changes in n-space and p-space; $S_1^q(V, M)$ is the subbundle of $J^q(V, M)$ induced by the inclusion $S_1^q \subset J^q$. Jet normal forms are given which show that whenever S_1^q is nonempty, then S_1^q either is the orbit of a single point if $n \leqq p$, or is the orbit of $[(n - p)/2] + 1$ distinct points if $n \geqq p$. The codimensions of S_1^q in J^q and the local equations of $S_1^q(f)$ are given. The proof of Theorem 1 for $n \geqq p$ is given in Section 3. The proof for the case $n < p$ is omitted since it parallels but is somewhat simpler than the proof for $n \geqq p$.

Suppose now that V and M are both n-dimensional manifolds, and that f maps V in M with rank $f_* \geqq n - 1$ everywhere. Further assume that $J^q(f)$ is transversal to the singularities $S_1^q(V, M)$ for all q. The object of Section 2 is to prove that under these conditions, the total characteristic class (Stiefel-Whitney class (mod 2) in the real case, and Chern class in the complex case) of V, $c(V)$, and the "pulled back" total characteristic class of M, $f^*c(M)$, are related by

$$c(V) = f^*c(M) - \textstyle\sum_{q=1}^{n} (j_q)_{\#}\, c(S_1^q(f)),$$

where j_q is the inclusion of $S_1^q(f)$ in V and $(j_q)_{\#}$ is the Gysin homomorphism of the cohomology of $S_1^q(f)$ into that of V.

Received September 21, 1962.

[1] This work was supported in part by the National Science Foundation.