DEFORMATIONS OF HOLOMORPHIC MAPPINGS

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The questions this paper seeks to answer are roughly the following:

(i) Given an arbitrary holomorphic mapping $f : M \to M'$ between fixed complex manifolds M, M' where M is compact, upon how many, if any, parameters can f depend?

(ii) If such parameters exist, what specifically are the deformations of the holomorphic mapping f?

We shall, under a cohomological restriction, answer (i) (Theorem 2); and, under the same cohomological restriction plus a restriction on f, give a solution to (ii) (Theorem 4). As examples show, the cohomological restriction is necessary; however, the restriction on f under (ii) is unsatisfactory and is probably unnecessary.

1.1. Let M and M' be two nonsingular complex manifolds of complex dimensions m and m' respectively, and assume that M is compact. Suppose that we are given a holomorphic mapping

$$f : M \to M'$$

and a complex space V with a distinguished point $v_0 \in V$.

DEFINITION 1. A deformation, with parameter space V, of the holomorphic mapping $f : M \to M'$ is given by a holomorphic mapping

$$g : M \times V \to M'$$

such that $g(m, v_0) = f(m)$ for all $m \in M$.

For each $v \in V$ we consider the holomorphic mapping

$$f_v : M \to M'$$

defined by $f_v(m) = g(m, v)$. Then $f_{v_0} = f$. Let $\{U_\alpha\}$, $\{U_j\}$ be open coordinate coverings of M, M' respectively, and let

$$Z_\alpha = (Z_\alpha^1, \cdots, Z_\alpha^m), \qquad W_j = (W_j^1, \cdots, W_j^{m'})$$

be holomorphic coordinates in U_α, U_j. The open sets $U_{\alpha,j} = U_\alpha \times U_j$ give a coordinate covering of $M \times M'$. If $g(U_\alpha \times V) \cap U_j \neq \emptyset$, then g is given as a mapping of $U_\alpha \times V$ to U_j by

$$W_j = g_{\alpha,j}(Z_\alpha, v) = (f_v)_{\alpha,j}(Z_\alpha), \tag{1.1}$$

and in particular

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