A CLASS OF GENERALIZED TI-GROUPS

BY

MICHAEL ASCHBACHER

This paper generalizes Suzuki's characterization of TI-groups. Specifically the following theorem is proved.

THEOREM. *Let G be a finite group in which the intersection of any two distinct Sylow 2-subgroups has 2-rank at most one. Let $O'(G)/O(G)$ be the smallest normal subgroup of $G/O(G)$ of odd index. Then $O'(G)/O(G)$ is one of the following:*

(1) *a 2-group,*

(2) *$GL_2(3)$, $SL_2(q)$, or the perfect nontrivial central extension of A_7 by a 2-group, or an extension of rank 1 of such a group,*

(3) *the extension of a 2-group by $L_2(q)$, $Sz(q)$, or $U_3(q)$, q even,*

(4) *the central product of two copies of $SL_2(5)$ with amalgamated centers, or its extension by an automorphism permuting the copies,*

(5) *$L_2(q)$, $q \equiv 3, 5 \bmod 8$, or $J(11)$, the smallest Janko group.*

The proof of the above theorem is a reasonably straightforward application of results of Alperin, Glauberman, and Shult on fusion, plus several classification theorems. The author would like to thank Professor John Walter for pointing out several errors in the original version of this paper.

1. kI-groups

Let G be a finite group. The *2-rank* $r(G)$ of G is the number of generators of an elementary 2-subgroup of G of maximal order if $|G|$ is even; if $|G|$ is odd, $r(G) = 0$. For k a nonnegative integer, we define G to be a *kI-group* if $r(G) > k$ and for any two distinct S_2-groups S and T of G, $r(S \cap T) \leq k$. If $k = 0$, G is a TI or "trivial intersection" group as defined by Suzuki [7].

The following elementary result is essentially Lemma 1 in [7].

LEMMA 1. *Let G be a kI-group. Then*

(1) *if $H \leq G$ with $r(H) > k$ then H is a kI-group,*

(2) *if H is a normal subgroup of odd order in G, then G/H is a kI-group.*

LEMMA 2. *Let G be a kI-group, S an S_2-group of G, $N = N_G S$ and $A \leq S$ such that either $r(A) > k$ or A is elementary of rank $k = 1$. Then*

$$\{A^g : g \in G, A^g \leq N\} = \{A^x : x \in N\}.$$

Proof. Alperin's theorem on fusion [1].

LEMMA 3. *Let G be a 2-nilpotent kI-group. Then either G is 2-closed or $r(G) = k + 1$.*

Received May 25, 1970.