# ON GENERALISED FREE PRODUCTS OF TORSION-FREE NILPOTENT GROUPS I

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## 1. Introduction

**1.1.** This note may be viewed as a continuation of the study of the generalised free products of two finitely generated nilpotent groups (see [1] and Joan Landman-Dyer [2]). Here we shall concern ourselves with the question of zero divisors in the group rings of such generalised free products.

**1.2.** We recall that a group $G$ is *indicable* if $G$ is either trivial or else has a homomorphism onto the infinite cyclic group. Hence, according to current terminology, $G$ is *locally indicable* if every finitely generated subgroup of $G$ is indicable (G. Higman [3]). In his important paper [3], G. Higman proved that the integral group ring of a locally indicable group has no zero divisors. Thus it is of interest to consider which groups are locally indicable. Until very recently the known locally indicable groups were those described in Higman's paper [3]. More specifically, in [3], Higman either proves or points out that ordered groups, poly-locally-indicable groups (see [4] for terminology) and ordinary free products of locally indicable groups are locally indicable. Latterly torsion-free groups with a single positive (i.e., no negative exponents occur in some) defining relation were added to the list [5] as well as every generalised free product of two locally indicable groups with a cyclic subgroup amalgamated (A. Karrass and D. Solitar [6]). Here, by mimicking part of the argument in [6], we shall prove the following

THEOREM. *Every generalised free product of any two finitely generated torsion-free nilpotent groups is locally indicable.*

It is worth pointing out that although the proof of Theorem 1 is not difficult the class of groups considered is quite complicated. For example there exists a generalised free product of two finitely generated torsion-free nilpotent groups which contains a non-trivial subgroup which coincides with its derived group (see [2]). For another illustration of the complicated nature of such generalised free products see [7].

## 2. The proof of Theorem 1

**2.1.** The key step in the proof of Theorem 1 is the following simple

LEMMA 1. *Let $A$ $(\neq 1)$ be an indicable group, $B$ a finitely generated torsion-free nilpotent group and let*

$$P = \{A * B;\, U\}$$

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