LOCALIZING CW-COMPLEXES

BY
D. W. ANDERSON

In the Sullivan-Quillen proof of the Adams conjecture on the image of the J-homomorphism, and in Sullivan's work on BPL and F/PL, it has become necessary to systematically exclude p-primary information about CW-complexes for certain primes p. The method of doing this is clear for loop spaces and for suspensions. Sullivan has used "local cells and local spheres" to do this for arbitrary complexes. Sullivan's construction suffers from the fact that it is not functorial, but is defined only up to homotopy type.

We give a construction below, inspired by the paper [2] of F. P. Peterson and the construction of Eilenberg-MacLane complexes given by E. Spanier in his book [3]. This construction is functorial, and has all of the desirable properties of Sullivan's construction.

1. R-groups

If R is a subring of the rational numbers containing 1, an abelian group A is called an R-group if the map $A = A \otimes Z \to A \otimes R$ is an isomorphism. Since all subrings of the rational numbers are free of torsion, $\mathrm{Tor}\,(A, R) = 0$. Thus the exact sequence

$$0 \to \mathrm{Tor}\,(A, R/A) \to A \otimes Z \to A \otimes R \to A \otimes R/Z \to 0$$

shows that A is an R-group if and only if $\mathrm{Tor}\,(A, R/Z) = 0$, $A \otimes R/Z = 0$.

Let $M = M(R) = \{m \mid m \text{ is an integer}, m \text{ a unit of } R\}$. Then $R = Z[M^{-1}]$, $M^{-1} = \{m^{-1} \mid m \in M\}$, so that M and R determine one another. Since M is countable, let $m_1, m_2, \cdots$ be an indexing of the elements of M. Let $R_i = Z$, $r_i: R_i \to R_{i+1}$ be multiplication by m_i. Then $R = \lim R_i$.

PROPOSITION 1.1. *If A is an R-group,* $\mathrm{Hom}\,(Z, A) = \mathrm{Hom}\,(R, A)$, $\mathrm{Hom}\,(R/Z, A) = 0$.

Proof. $\mathrm{Hom}\,(R, A) = \lim\mathrm{inv}\,(\mathrm{Hom}\,(R_i, A))$. If A is an R-group, it is an R-module, so that $m_i : A \to A$ is an isomorphism. Thus every map in the inverse system $\mathrm{Hom}\,(R_i, A)$ is an isomorphism, so that $\mathrm{Hom}\,(R, A) = \mathrm{Hom}\,(R_1, A) = \mathrm{Hom}\,(Z, A)$. Thus $\mathrm{Hom}\,(R/Z, A) = 0$.

PROPOSITION 1.2. *If A is an R-group,* $\mathrm{Ext}\,(R, A) = 0 = \mathrm{Ext}\,(R/Z, A)$.

Proof. From the fact that $\mathrm{Hom}\,(R, A) \to \mathrm{Hom}\,(Z, A)$ is surjective, and $\mathrm{Ext}\,(Z, A) = 0$, we see that $\mathrm{Ext}\,(R/Z, A) = \mathrm{Ext}\,(R, A)$.

Let $F_0 \to F_1 \to R/Z$ be a free resolution of R/Z. Since R is an R-group,

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