ON A CLASS OF DOUBLY TRANSITIVE GROUPS

BY
ERNEST SHULT

The purpose of this paper is to prove the following theorem:

THEOREM. *Let G be a transitive group of permutations on the (finite) set of letters Ω. Let G_α be the subgroup of G fixing the letter α in Ω. Suppose G_α contains a normal subgroup Q of even order, which is regular on $\Omega - (\alpha)$. Then either*

(a) *G is a subgroup of the group of semi-linear transformations over a near field of odd characteristic or*

(b) *G is an extension of one of the groups $SL(2, q)$, $Sz(q)$ or $U(3, q)$ by a subgroup of its outer automorphism group. ($|\Omega| = 1 + q$, $1 + q^2$ or $1 + q^3$ in these three respective cases ($q = 2^n$).)*

Essentially "half" of this theorem was proved by Suzuki [8], under the assumption that the quotient group G_α/Q had odd order. We therefore consider only the case that G_α/Q has even order.

Since Q is regular on $\Omega - (\alpha)$, we may express G_α as a semidirect product $G_{\alpha\beta} Q$ where $G_{\alpha\beta} = G_\alpha \cap G_\beta$, the subgroup of permutations fixing both α and β.

For the rest of this paper, all groups considered are finite. We write $|X|$ for the cardinality of set X. If X is a subset of a group G, we write $X \subseteq G$, and if X is a subgroup of G, we write $X \leq G$. If $X \subseteq G$, $\langle X\rangle$ will denote the subgroup of G generated by X. If X is a subset of G, X^G denotes the set of all conjugate sets $\{g^1Xg \mid g \in G\}$. We will frequently write $\langle X^G\rangle$ instead of the more cumbersome $\langle \bigcup_{Y \in X^G} Y\rangle$. This is the normal closure of X in G and represents the smallest normal subgroup of G containing X. If M is a group of (right) operators of a group G it will frequently be convenient to proceed with computations in the semi-direct product GM and also to view GM as a group of right operators of G, the elements of G acting by conjugation. Action of these operators is indicated by exponential notation. Thus if $x \in G$, $g^{-1}xg$ may be written x^g and if σ is an automorphism of G, we may write

$$(x^g)^\sigma = x^{g\sigma} = x^{\sigma \cdot g^\sigma}.$$

The commutator $x^{-1}y^{-1}xy$ is written $[x, y]$. If σ is an automorphism of G and if $x \in G$, then the commutator $[x, \sigma]$ is assumed to be computed in the semidirect product $G\langle\sigma\rangle$, so $[x, \sigma] = x^{-1} \cdot x^\sigma$. If π is a set of primes, a π-group is a group whose order involves only primes in π. As usual, π' denotes the complement of π in the set of all primes. If π consists of a single prime p, the symbol p (rather than $\{p\}$) may replace the symbol π in the notation of

Received March 30, 1970.