ON ANALYTIC STRUCTURE IN THE MAXIMAL IDEAL SPACE OF $H_\infty(D^n)$

BY

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Let $H_\infty(D^n)$ denote the complex Banach algebra of bounded holomorphic functions on the open unit polydisc

$$D^n = \{(z_1, \cdots, z_n) \in \mathbf{C}^n : |z_1| < 1, \cdots, |z_n| < 1\}.$$

The map $(z_1, \cdots, z_n) \to f(z_1, \cdots, z_n)$ imbeds D^n as an open subset of the maximal ideal space of $H_\infty(D^n)$; so we let $M(H_\infty(D^n))$ denote the closure of D^n in this space. By an analytic map into $M(H_\infty(D^n))$ we mean a function

$$F : D^m \to M(H_\infty(D^n))$$

such that $\hat{f} \circ F$ is analytic in D^m for every f in $H_\infty(D^n)$, where $\hat{f}$ is the Gélfand extension of f to $M(H_\infty(D^n))$. The image of F is called an analytic set in $M(H_\infty(D^n))$. If F is one-one, then $F(D^n)$ is a m-dimensional analytic polydisc.

In this paper we construct various dimensional analytic polydiscs in $M(H_\infty(D^n))$ as limits of analytic maps into D^n and compare these in a natural way with the analytic structure in $M(H_\infty(D))^n$, the n-fold Cartesian product of $M(H_\infty(D))$. We also show that only points belonging to the closure of zero sets of functions in $H_\infty(D^n)$ can belong to analytic sets obtained in this manner.

The maximal ideal space of the algebra $H_\infty(D)$ has been extensively studied, beginning with I. J. Schark [13], and continuing with D. Newman [12], A. Gleason and H. Whitney [5], L. Carleson [3, 4], A. Kerr-Lawson [11], K. Hoffman [8, 10], and others. In the paper of I. J. Schark, it was shown that there exist non-trivial analytic mappings from D into $M(H_\infty(D))\backslash D$. Angus Kerr-Lawson [11] extended the Schark idea and showed that "non-tangential" and "oricycular" points in $M(H_\infty(D))$ lie in non-trivial analytic sets. By an algebraic argument, K. Hoffman [8] showed that each non-trivial Gleason part in $M(H_\infty(D))$ is a 1-dimensional analytic disc. Shortly thereafter Professor Hoffman [10] gave a "geometric" method for obtaining the coordinate maps for the analytic discs in $M(H_\infty(D))$.

The natural inductive vehicle for generalization to higher dimensional polydiscs is the topological tensor product $\otimes_\lambda^n H_\infty(D)$, where $\otimes_\lambda^n$ is the completion of the algebraic tensor product $\otimes^n$ in the uniform norm. However, it is now well known (see [1]) that $\otimes_\lambda^n H_\infty(D) \neq H_\infty(D^n)$. Hence, the lifting of 1-dimensional results becomes more than routine.

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