

SECONDARY OPERATIONS IN K -THEORY AND APPLICATIONS TO METASTABLE HOMOTOPY

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1. Introduction

There has been considerable research into the general properties of secondary and higher order cohomology operations in K -theory [1], [6]. Most of this has been directed towards the construction of a K -theoretic version of the Adams spectral sequence.

In the present paper we calculate explicitly with particular operations to prove the following two theorems:

THEOREM I. *Let $n + 1 \equiv 2^r (2^{r+1})$, let $n > 0$ and let $\theta \in \pi_{4n-k}(S^{2n-k})$ be such that $E^{k+1}\theta = [\iota_{2n+1}, \iota_{2n+1}]$. Then:*

- (a) $2^q \mid o(\theta)$ where $q = \min \{ (k+2)/2, n-r-1 \}$.
- (b) If $r = 0$ and $n \neq 2, k = 0$.
 If $r \neq 2 (4), r > 0$, and $n \neq 5, 7, k \leq 2r + 1$.
 If $r \equiv 2 (4), k \leq 2r + 2$.
 If $n = 7, k \leq 9$.
 If $n = 5, k \leq 5$.
 If $n = 2, k \leq 1$.

THEOREM II. *Suppose $\gamma \in \pi_{4n+8j-2}(S^{2n}), H(\gamma) = \rho_j, 8j < 2n - 3, 2^r \mid n + 4j$ and $r \geq 1$. Then $2^{r+2-k} \mid o(E^{2k}\gamma)$.*

Part (b) of Theorem I is equivalent to a theorem about k -frames on S^n which differs from the best possible result, due to Adams [2], in that the condition $k \leq 2r$ should be replaced by $k \leq 2r - 1$ if $r \equiv 0, 1 \pmod{4}$ and the exceptions for $n = 2, 5, 7$ should be removed. Our proof differs from Adams' in that it is essentially unstable and independent of the topology of stunted projective spaces. It seems worth noting that Theorem Ib is equivalent to the best result obtainable by Adams' method without the use of KO -theory except for the cases $n = 2, 5, 7$.

Theorem II is of interest when r is large compared to j . In these cases Mahowald's results on metastable homotopy [5] leave open the essentiality of $[\iota_{2n-1}, \rho_j]$.

Theorems I and II are both applications of our principal result, Theorem 5.3, which characterizes desuspensions of $[\iota_{2n+1}, \iota_{2n+1}]$ in terms of the operations defined in §2.

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