ON THE CHAIN-COMPLEX OF A FIBRATION

BY

V. K. A. M. GUGENHEIM[1]

It was proved in [7] that a Serre-fibration $\pi : E \to B$ with fibre F can be replaced, as far as its singular complex is concerned, by a "twisted Cartesian Product" $B \times_\xi F$. In [1], [2], [9], [10] it was shown how, from this, the theorem of E. H. Brown on the structure of a suitable differential on $B \otimes F$ could be derived. Here, as throughout the present paper we use the same letter to denote a space, its singular complex and its *normalised* chain-complex.

At the time of these papers, the relevant algebraic ideas—in particular that of a coalgebra, a comodule and the cotensor product—were not well understood; due to this both the proofs given and the nature of the result obtained remained obscure. We hope to clarify these matters here.

The existence of the differential itself is established by a simple "perturbation argument", Chapter 3; essentially the same argument appears in [10]. Then, the B-comodule structure and the dual A-module structure, where A denotes the group of the twisted Cartesian product $B \times_\xi F$, are investigated in Chapter 4. Here we follow the method of Weishu Shih [1]. The form of the differential given by E. H. Brown then follows from a simple, purely algebraic lemma, Chapter 2; also it follows that the appropriate chain complex for the fibration E' induced by a map $\beta : B' \to B$ is the co-tensor product $B' \otimes^B (B \times_\xi F)$.

The result of Eilenberg and Moore, [4], namely $H(E') = \mathrm{Cotor}^B\ (B', E)$ is now not hard to prove. We prove it here assuming only—as was done in [6]—that the action of $\pi_1(B)$ an the homology of F is trivial. Chapter 5 merely summarises the necessary cohomological algebra from [4] and [6].

A point of notation: the symbol of any object also stands for the identity map on that object.

I am indebted to several conversations with John Moore.

1. Preliminaries

Let R be a commutative ring with unit; C^+ denotes the category of positive complexes over R, i.e. the sequences

$$\to K_{n+1} \xrightarrow{d_{n+1}} K_n \xrightarrow{d_n} K_{n-1} \to \cdots \to K_1 \to K_0 \to 0$$

of R-modules and R-morphisms such that $d_n d_{n+1} = 0$. There is an evident forgetful functor $\square : C^+ \to G^+$, the category of positive graded R-modules;

Received March 6, 1970.

[1] The author was partially supported by a National Science Foundation grant.