ON THE STRONG LIFTING PROPERTY

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I. Introduction

Let X be a locally compact Hausdorff space, μ a positive Radon measure on X of support X. It is known that there is a strong lifting (definition below) for (X, μ) if X is metrizable [6, Ch. VIII, Theorem 3] or if X is a locally compact group and μ a Haar measure [5] or if X is a product of metrizable spaces and μ the product of measures on them [6, Ch. VIII, No. 2], or if μ is absolutely continuous with respect to any such measure [2].

Those are virtually the only cases for which the existence of a strong lifting is known, despite the interest a general existence theorem has for the disintegration theory of measures [3], [4], [6].

As it seems too difficult at present to decide whether or not there is always a strong lifting for (X, μ)—this might be an indecidable problem for all that is known—the next thing to do is to find measures admitting strong liftings. It is shown here in this context that one can generate new measures admitting strong liftings from known ones by taking sums, multiples, infima and suprema. The main result obtained can be summarized saying that the set $L(X)$ of Radon measures μ on X such that $|\mu|$ admits an almost strong lifting (definition below) forms a band in the complete vector lattice $M(X)$ of all Radon measures on X.

We note in passing that the desired result $M(X) = L(X)$ for arbitrary X is true if and only if it holds in case that X is any product of unit intervals [2]. Using the fact that $L(X)$ is a band in $M(X)$ one obtains the following reduction of the general existence problem.

THEOREM. *$L(X) = M(X)$ for all X if and only if for every product Π of unit intervals and every positive Radon measure ν on Π there is a Radon measure $\mu > 0$ on Π such that μ is absolutely continuous with respect to ν and admits an almost strong lifting.*

It is clear that the condition is necessary. If it is satisfied, on the other hand, then the band complementary to $L(\Pi)$ is zero for all Π, hence $L(\Pi) = M(\Pi)$ for all Π, hence $L(X) = M(X)$ for all X.

II. Notation and conventions

X is a set and $(X, \mathfrak{F}, \mu)$ a measure space on X with σ-algebra $\mathfrak{F}$ and positive measure μ on $\mathfrak{F}$, strictly localizable and equal to its Carathéodory extension. $\bar{\mu}$ is the associated essential integral, μ^*, $\bar{\mu}^*$ are the corresponding outer measures (upper integrals). A μ-null function or set is one which is locally negli-

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