A CHARACTERIZATION OF SOME SPECTRAL MANIFOLDS FOR A CLASS OF OPERATORS[1]

BY

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Introduction

In this paper we shall characterize certain spectral manifolds for a class of bounded linear operators acting on a complex Banach space. Each operator T of the class has a real spectrum $\sigma(T)$ and its resolvent operator $R(\zeta; T) = (\zeta I - T)^{-1}$ satisfies an *n-th* order rate of growth (G_n) near $\sigma(T)$ in the sense that

$$(G_n) \qquad \begin{array}{ll} |\operatorname{Im} \zeta|^n \| R(\zeta; T) \| \leq K & \text{for} \quad 0 < |\operatorname{Im} \zeta| < 1, \\ |\operatorname{Im} \zeta| \| R(\zeta; T) \| \leq K & \text{for} \quad 1 \leq |\operatorname{Im} \zeta|. \end{array}$$

This characterization will be as the null spaces (kernels) of certain bounded operators constructed from T by means of contour integrals. Bounded operators satisfying (G_n) were studied by R. G. Bartle [1], [2] and, independently, unbounded operators satisfying this condition were studied by the author [6]. Under additional assumptions, each operator of the class has a spectral decomposition similar to that of a self-adjoint transformation (cf. [2] or [6]).

For each bounded operator T satisfying the condition (G_n) and for each closed subset $F \subset R$ let $\mathbf{X}(F)$ denote the closed linear manifold of all vectors x whose local spectra relative to T lie in F. In §1 we review properties of operators $K(a, b)$ studied in [6] and introduced by E. R. Lorch [7] for self-adjoint operators. In §2 we introduce for each $t \in R$ operators $H_-(t)$ and $H_+(t)$ and derive their basic properties. We shall prove that $\mathbf{X}((-\infty, t])$ is the kernel of $H_+(t)$, that $\mathbf{X}([t, +\infty))$ is the kernel of $H_-(t)$, and that $\mathbf{X}([a, b])$ is the kernel of $(T - aI)^n(T - bI)^n - K(a, b)$. This characterizes $X(F)$ for any closed interval F for each T of the class. These results strengthen similar results in [6] where the author assumed that T lacked a point spectrum and then, at a later stage, assumed that T had a purely continuous spectrum. In §3, with additional hypotheses we shall obtain a spectral decomposition of T in terms of the kernel of $H_+(t)$ and the closure of its range. These manifolds yield a closed resolution of the identity for T in the sense of F. J. Murray [8]. The section concludes with a generalization of a form of the spectral theorem for self-adjoint transformations which is applied to obtain the classical integral representation for bounded self-adjoint operators.

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