COMPOSITION SERIES FOR SIMPLEX SPACES

BY

ALAN GLEIT[1]

A general theory of composition series was given in [3]. It was there applied to the case of separable simplex spaces. The authors characterized the separable GC-spaces and partially characterized the separable GM-spaces. We shall, here, generalize and extend those results.

The notations and definitions are those used in [1], [2], [3], [4]. V will always denote a simplex space. For a set $A \subseteq P_1(V)$, we let $\bar{A}$ or A^- be the weak* closure of A and $A^+ = A - \{0\}$, with the exception that $E^+ = EP_1(V)^+$. For $q \in P_1(V)$, we shall denote by π_q the unique maximal probability measure with resultant q. If V is separable, then π_q is supported by $EP_1(V)$.

Let X be any topological space and p any topological property. If a subset $G \subseteq X$ has property p, we write $G \subseteq_p X$ and say that G is a *p-subset* of X. (For a full account, see [3, §3].) A property p is *inductive* if for each nonempty closed set F and each open G in X we have: $G \subseteq_p X$ implies that $G \cap F \subseteq_p F$. We say a property p is *strongly inductive* if (1) p is inductive and, given G_1, G_2 open, F closed in X, we also have:

(2) $G_1 \subseteq G_2 \subseteq X$ and $G_1 \subseteq_p X$ imply $G_1 \subseteq_p G_2$.

(3) $G_1 \subseteq G_2 \subseteq_p X$ implies $G_1 \subseteq_p X$.

(4) $G_1 \subseteq_p F \subseteq X$ implies $G_1 \subseteq_p X$.

For $X = \max V$, we shall consider the following properties:

(C) $G \subseteq_C \max V$ means that elements of V restrict to continuous functions on G.

(M) $G \subseteq_M \max V$ if each net in G which converges to a point of G converges to no other point of $\max V$.

(n) (for $n \geq 2$) $G \subseteq_n \max V$ if each sequence in G which converges to a point of G converges to at most n points in $\max V$.

PROPOSITION 1. *The properties* (C), (M) *and* (n) *are strongly inductive.*

Proof. That (C) and (M) are strongly inductive is shown in [3, Prop' 4.3]. That (n) is strongly inductive is obvious.

If J is a closed ideal in V, we let $P_1(J)$ be the positivestates of J and $EP_1(J)$ be the pure states of J when we consider J as a simplex space in its own right.

Received February 9, 1970.

[1] Parts of this paper are from my Ph.D. thesis submitted to Stanford University, written under the direction of E. Effros. Partially supported by a National Science Foundation development grant while at Carnegie-Mellon University.