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THE CUSP AMPLITUDES OF THE CONGRUENCE SUBGROUPS OF THE CLASSICAL MODULAR GROUP (II)

BY

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1. Introduction

The homogeneous modular group ${}_1\Gamma = SL(2, Z)$. If $A \in {}_1\Gamma$ and

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then A induces the linear fractional substitution $z \to A(z)$, where

$$A(z) = (az + b)/(cz + d), z = x + iy,$$

where x and y are real numbers. The group of all substitutions is known as the inhomogeneous modular group. A matrix $A \neq \pm I$, where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

and the substitution $A(z)$ are called parabolic if for a rational number ζ or $\zeta = \infty$, $A(\zeta) = \zeta$. We call ζ the fixed point of $A(z)$ and of A. For a parabolic matrix P with fixed point ζ there exist $B \in {}_1\Gamma$ and a rational integer $n \neq 0$ such that

$$P = \pm B^{-1}U^nB \quad \text{where } U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } \zeta = B^{-1}(\infty).$$

The modulus $|n|$ of n is called the amplitude of P. If Γ is a subgroup of ${}_1\Gamma$ and $P \in \Gamma$ then ζ is also referred to as a fixed point or a cusp of Γ. The cusp amplitude of ζ in Γ is the smallest positive rational integer k such that

$$\pm B^{-1}U^kB \in \Gamma.$$

Two cusps η and ζ are said to be eqivalent under Γ, for which we write $\eta \sim_\Gamma \zeta$, if there is a $A \in \Gamma$ such that $\eta = A(\zeta)$. Equivalent cusps in Γ have the same amplitudes. For $\Gamma \subset {}_1\Gamma$ we denote by $C(\Gamma)$ the subset of the set of all positive rational integers containing all different cusp amplitudes of Γ.

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