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RESIDUAL NILPOTENCY OF FUCHSIAN GROUPS[1]

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0. Introduction

If we have a group with a relatively complicated structure, but with many homomorphisms into groups of a simpler nature, the study of these homomorphisms may enable us to obtain information not easily available otherwise about the original group. If the family of groups which are targets of the homomorphisms is characterized by a property P, the information we obtain is in theory complete if the original group is "residually P" in Philip Hall's terminology; that is to say, if there are enough homomorphisms to groups with the property P to distinguish any one element of the group from any other, so that homomorphisms to groups with property P provide a sort of "coordinate system".

In the case of Fuchsian groups, there is an additional motivation for studying homomorphisms to different classes of group—the homomorphic images can always be realized as groups of automorphisms of Riemann surfaces. Homomorphisms of Fuchsian groups into finite groups, finite cyclic groups, finite abelian groups and finite soluble groups have been studied, with perhaps a disproportionate amount of attention, for which I must admit some personal responsibility, being paid to the very special and certainly fascinating class of Riemann surfaces for which Hurwitz's maximum of $84(g - 1)$ automorphisms is attained (see [1], [3], [4], [5], [8], [10], [16], [18], [19], [20], [21], [22], [24]).

All Fuchsian groups are residually finite, and many are residually finite-and-soluble (see [24]).

In his study of residual solubility, Sah was led to introduce the "p-periods" of a Fuchsian group, which can be regarded as the first step in the direction of the p-localization introduced in the present paper. Our object here is to close the obvious gap in the list of target groups for homomorphisms by studying maps into finite nilpotent groups. We restrict attention to co-compact Fuchsian groups, so that we can use the technique of Gruenberg [9] based on Hirsch's theorem [12] that a finitely generated nilpotent group is residually finite. Therefore a finitely generated group is

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