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AN HARMONIC ANALYSIS FOR OPERATORS: F. AND M. RIESZ THEOREMS

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Dedicated to the memory of Professor Karel De Leeuw

1. Introduction

De Leeuw introduced and studied an harmonic analysis for operators in [2] and [3]. With each bounded linear operator T on a homogeneous Banach space B on the circle group $\mathbf{T}$, he associated a formal Fourier series,

$$(1.1) \qquad T \sim \sum_{-\infty}^{\infty} \pi_n(T)$$

In [2], he established formal properties of the series (1.1) and remarked that analogues of all the results of Sections 2–5 of [2] are valid for any compact abelian group. In Section 6 of [2], De Leeuw obtained an extension to operators of the first theorem of F. and M. Riesz. This classical theorem asserts that a measure whose Fourier Stieltjes transform vanishes on the negative integers must be absolutely continuous. In this paper we obtain an extension of the above theorem to operators on certain homogeneous Banach spaces on a compact abelian group with ordered dual.

In [3], De Leeuw restricted himself to the case where B is $L^2(\mathbf{T})$. He extended the notions of support and analyticity to operators and established an analogue for operators of the second theorem of F. and M. Riesz. This theorem asserts that a function having all its negative Fourier coefficients zero, cannot vanish on a subset of $\mathbf{T}$ having positive Lebesgue measure, unless it is zero almost everywhere. In this paper we extend the notions of support and analyticity to operators on B, where B is $C(G)$ or one of the $L^p(G)$, $1 \leqslant p < \infty$, and G is a compact abelian group with ordered dual. We use an example due to De Leeuw and Glicksberg [4] to show that a natural generalization of the second F. and M. Riesz theorem for operators on $L^2(G)$ fails, unless some condition is imposed on the ordering of Γ, the dual group of G. The condition we impose is motivated by an extension of the classical theorem of F. and M. Riesz to measures on a compact abelian group with ordered dual due to De Leeuw and Glicksberg in [4]. Our generalization of the second theorem of F. and M. Riesz for

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