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ON GRAPHS WITH EDGE-TRANSITIVE AUTOMORPHISM GROUPS

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In [4], Goldschmidt considered groups G with finite subgroups M_1 and M_2 and the following three properties:

(i) $G = <M_1, M_2>$.

(ii) No non-trivial normal subgroup of G is contained in $M_1 \cap M_2$.

(iii) $|M_i/M_1 \cap M_2| = 3$ for $i = 1,2$.

He was able to give the exact structure (the isomorphism classes) of all possible pairs of subgroups M_1 and M_2. In his proof he used a graph theoretical approach:

Any group G with properties (i) and (ii) operates as an edge-transitive group of automorphisms on a graph Γ whose vertex set is

$$\{M_1 x / x \in G\} \dot{\cup} \{M_2 x \mid x \in G\}$$

and where two vertices are adjacent iff they have non-empty intersection. G operates on Γ by right multiplication, the vertex-stabilizers in G are conjugate to M_1 or M_2, and the edge-stabilizers are conjugate to $M_1 \cap M_2$ (see [4, (2.6)]).

Since G is a homomorphic image of the amalgamated product of M_1 and M_2 with respect to $M_1 \cap M_2$, one can study this amalgamated product and the corresponding graph Γ. Serre [9] has shown in this case that Γ is a tree. Hence the above problem leads to the investigation of edge-transitive groups of automorphisms of the trivalent tree with finite vertex-stabilizers.

We use this method to investigate a more general situation. We make the following hypotheses.

Hypothesis A. Let G be a group and M_1 and M_2 be finite subgroups of G such that:

(1) $G = <M_1, M_2>$.

(2) No non-trivial normal subgroup of G is contained in $M_1 \cap M_2$.

(3) $|M_i/M_1 \cap M_2| = 2^{n_i} + 1$, $n_i \geq 1$, $i = 1,2$ and $\max\{n_1, n_2\} > 1$.

(4) There exists a normal subgroup N_i in M_i such that

$$N_i/R \simeq L_2(2^{n_i})' \quad \text{for } R = \cap_{x \in M_i} (M_i \cap M_j^x) \text{ and } \{i,j\} = \{1,2\}.$$

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