ON REALIZING NAKAOKA'S COINCIDENCE POINT TRANSFER AS AN S-MAP

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1. Introduction

In [6] the existence was asserted of a transfer for fibre bundles whose fibers were compact manifolds. This transfer is a homomorphism in singular homology or cohomology. In [1] this transfer was shown to be induced by an S-map, at least for fibre bundles with compact Lie groups as structure groups and finite complexes as base spaces. When a transfer is induced by an S-map then it exists in every cohomology and homology theory. In the case of the transfer in [1] the S-map led immediately to a simple topological proof of the Adams Conjecture.

The transfer map for finite covering spaces was known to be induced by an S-map. This fact was crucial in the proof of the celebrated Kahn-Priddy theorem [8].

At present there are several different transfers defined in various circumstances. Many are defined only for singular homology groups. Sometimes these are not realized by S-maps. For example the transfer for ramified coverings [10] does not commute with the Steenrod algebra [7] (also attributed to Dold). Hence it cannot be realized by an S-map. Recently, Ralph Cohen [4] has shown that the ramified covering transfer can be realized if the ramified covering is localized at certain primes. This was also known to Larry Smith.

In [9], Nakaoka obtained a transfer in the following situation: Let

$$E_1 \xrightarrow{p_1} B$$
 and $E_2 \xrightarrow{p_2} B$

be fibre bundles with closed oriented m-dimensional manifolds M_1 and M_2 as fibres. Suppose that $\pi_1(B)$ acts trivially on both $H^m(M_1; \mathbb{Z})$ and $H^m(M_2; \mathbb{Z})$. Let f and g: $E_1 \rightarrow E_2$ be fibre preserving maps covering the identity 1: $B \rightarrow B$. Then there is a homomorphism

$$\tau_{f,g}: H^*(E_1; \mathbb{Z}) \to H^*(B; \mathbb{Z})$$

such that $\tau_{f,g} \circ p_1^*$ is multiplication by $\lambda(\bar{f}, \bar{g})$ where \bar{f} and \bar{g} are the restrictions

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