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ON SPECTRAL DECOMPOSITION OF CLOSED OPERATORS ON BANACH SPACES

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This paper is concerned with presenting some necessary and sufficient conditions for a closed operator to have the spectral decomposition property. We refer to [2] for notations and terminology, but for convenience we repeat some definitions.

Throughout this paper, T is a closed operator with domain D_T and range in a Banach space over the complex field **C**. Let **N** denote the set of natural numbers and let $Z^+ = \mathbf{N} \cup \{0\}$. If S is a set then $\bar{S}$ is the closure, S^c is the complement, $\operatorname{Int} S$ is the interior and we denote by $\operatorname{cov} S$ the collection of all finite open covers of S. Without loss of generality, we assume that for $S \subset \mathbf{C}$, every $\{G_i\}_{i=0}^n \in \operatorname{cov} S$ has, at most, one unbounded set G_0. A set $S \subset \mathbf{C}$ is said to be a neighborhood of ∞, in symbols $S \in V_\infty$, if $\overline{S^c}$ is compact in **C**. Given T, $\sigma(T)$ is the spectrum, $\sigma_a(T)$ is the approximate point spectrum, $\rho(T)$ is the resolvent set and $R(\cdot; T)$ is the resolvent operator. If A is a bounded operator then $\rho_\infty(A)$ denotes the unbounded component of $\rho(A)$. If T has the single valued extension property (SVEP), then $\sigma_T(x)$, $\rho_T(x)$ and $x(\cdot)$ denote the local spectrum, the local resolvent set and the local resolvent function, respectively, at $x \in X$.

For $S \subset \mathbf{C}$, we shall make an extensive use of the spectral manifold

$$X(T, S) = \{x \in X: \sigma_T(x) \subset S\}. \tag{1}$$

We write $\operatorname{Inv} T$ for the lattice of the subspaces of X which are invariant under T. For $Y \in \operatorname{Inv} T$, $T|Y$ is the restriction of T to Y and $\hat{T} = T/Y$ denotes the coinduced operator by T on the quotient space X/Y. The coset $\hat{x} = x + Y$ is a vector in X/Y and $\hat{x} \in D_{\hat{T}}$ iff $\hat{x} \cap D_T \neq \varnothing$. If f is an X-valued function then the function $\hat{f}$ has the range in X/Y.

1. Introduction

In this section, certain basic notions pertaining to the spectral theory will be touched upon and some preliminary results will be established to be used in

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