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ON THE TENSOR PRODUCT OF A CLASS OF NON-LOCALLY CONVEX TOPOLOGICAL VECTOR SPACES

BY

W. DEEB AND R. KHALIL

0. Introduction

Let f be a real valued function defined on $[0, \infty)$ which satisfies:
(i) $f(x) = 0$ if and only if $x = 0$;
(ii) f is increasing;
(iii) $f(x + y) \leq f(x) + f(y)$;
(iv) $\lim_{x \to 0^+} f(x) = 0$.
It is clear that every such function is continuous. For every sequences $x = (x_n)$ we define

$$|x|_f = \sum_{n=0}^{\infty} f|x_n|.$$

The space $L(f)$ is the set of all real sequences $x = (x_n)$ such that $|x|_f < \infty$. One can easily show that $|x|_f$ defines a metric on $L(f)$.

It was shown in [1] that $(L(f), |\ \ |_f)$ is a complete metric space.

The space $(L(f), |\ \ |_f)$ is a topological vector space [1]. For more about $L(f)$ spaces we refer to [2], [3], [7]. The object of this paper is to characterize the isometries of $(L(f), |\ \ |_f)$ and to define the projective tensor product of $L(f)$ with itself, proving some results on the tensor product.

Throughout this paper, N will denote the set of positive integers. If X and Y are topological vector spaces, $WL(X, Y)$ will denote the weakly continuous linear operators from X into Y, and $B(X, Y)$ the continuous bilinear functional on $X \times Y$. The dual of a topological vector space X will be denoted by X^*.

1. Isometries of $L(f)$

A continuous linear operator F: $L(f) \to L(f)$ will be called an isometry if

$$|F(x)|_f = |x|_f \quad \text{for all } x \in L(f).$$

Let e_i be the sequence with 1 at the ith-coordinate and zero elsewhere.

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