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A CRITERION FOR DELOOPING THE FIBRE OF THE SELF-MAP OF A SPHERE WITH DEGREE A POWER OF A PRIME

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Fix, once and for all, p to be an odd prime, and n and j to be strictly positive integers. Let F be the homotopy fibre of the self-map of S^{2n-1} of degree p^j (i.e.,

$$F \to S^{2n-1} \xrightarrow{p^j} S^{2n-1}$$

is a fibration up to homotopy). Notice that F is its own localization at p. The sphere S^{2n-1} itself, localized at p, deloops if and only if n divides $p-1$. In [2], the second author showed that for certain values of p, n and j, the fibre F deloops. The deloopings are of the form $BG(\mathbf{F}_q)^+_{(p)}$ where $G(\mathbf{F}_q)$ is the universal Chevalley group of some exceptional Lie type over the finite field $\mathbf{F}_q$, q a power of a prime different from p. Here "+" denotes Quillen's "plus construction" (see [6]) and (p) denotes localization at the prime p. In all these cases n divides $p-1$. The main result of this paper is the following more general theorem:

THEOREM I. *F is a loop space if and only if n divides $p-1$.*

We divide the paper into two, essentially separate, parts. In the first part, when n divides $p-1$ we give two methods for constructing a delooping. One of these deloopings is of the form $(BG^+)_{(p)}$ where G is the special linear group of a finite field. In the second part we show that if a delooping exists, then n divides $p-1$.

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