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A BACKWARD HARNACK INEQUALITY AND FATOU THEOREM FOR NONNEGATIVE SOLUTIONS OF PARABOLIC EQUATIONS

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Introduction

It is not an uncommon happening in the development of elliptic and parabolic p.d.e. that resolution of a problem first appears in the elliptic case and shortly after there is an attempt to adapt the techniques to the corresponding parabolic problem. In the majority of cases the adaptation succeeds with relative ease; but when it does not succeed so readily, or even not at all, a new and hopefully interesting insight into solutions of the parabolic problem is needed.

Such is the case in the study of the classical Fatou theorem for solutions, $u(x, t)$, of a parabolic partial differential equation of the form

$$Lu(x, t) \equiv \sum_{i, j=1}^{n} D_{x_i}\left(a_{ij}(x, t) D_{x_j} u(x, t)\right) - D_t u(x, t) = 0.$$

In particular, we consider solutions, u, defined in the cylinder $D_+ \equiv Dx(0, \infty)$, $D \subset \mathbf{R}^n$, which are nonnegative there and we want to study their pointwise boundary behavior, especially at points on the lateral boundary, $S_+ \equiv \partial Dx(0, \infty)$.

The assumptions on the operator L and domain $D \subset \mathbf{R}^n$ are as follows:

(i) The matrix $(a_{ij}(x, t))$ is bounded, measurable, symmetric, and uniformly positive definite, i.e., there exists $\lambda > 0$ such that for all $x \in \mathbf{R}^n$, $\xi \in \mathbf{R}^n$ and $t > 0$,

$$\lambda|\xi|^2 \leq \sum_{i, j=1}^{n} a_{ij}(x, t)\xi_i\xi_j \leq (1/\lambda)|\xi|^2.$$

(ii) D is a bounded Lipschitz domain in $\mathbf{R}^n$.

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