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DECOMPOSITIONS THAT DESTROY SIMPLE CONNECTIVITY

BY

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We shall be concerned with a monotone decomposition of R^3 with only one nondegenerate decomposition element X. We use g to denote the decomposition map and $g(R^3)$ the decomposition space. Also, D denotes a disk. To determine if $g(R^3)$ is simply connected we shall be concerned with whether maps of Bd D into $g(R^3)$ can be extended to D.

At the Summer Institute on Set Theoretic Topology at Wisconsin in 1955 I gave a talk entitled "What topology is here to stay" in which I envisioned decompositions of R^3 as a very viable area for research. I mentioned R.L. Moore's monotone decomposition theorem [3] for S^2 which states that if G is a nondegenerate upper semicontinuous decomposition of S^2 each of whose elements is a continuum that does not separate S^2, then the decomposition space is S^2. I pointed out that the theorem was false if one replaced S^2 by S^3 and gave as an example the decomposition whose only nondegenerate element is a circle. The earlier version of the Summary of Lectures and Seminars [1] reported on page 26 that the reason I gave that the decomposition space differed from S^3 was that it *is not simply connected*. The second printing of [1] made the correction by replacing the *is not simply connected* part of the statement by *does not remain simply connected* on the removal of some point. It was also claimed there and in [2] that the decomposition space of S^3 (or R^3) whose only nondegenerate element is a solenoid is not simply connected. When I was assembling copies of my publications it was called to my attention that a proof of this claim had not been published. It is the purpose of this paper to fill that gap. Other claims were made in [2] about the simple connectivity of other monotone decompositions (perhaps with many nondegenerate elements) of R^3, but we shall not treat them in this paper.

Richard Skora read an early draft of this paper and made valuable suggestions for improving some proofs.

1. X is a standard solenoid

In this case X is the intersection of smooth unknotted tori $T_1, T_2, \ldots$ where T_{i+1} winds around T_i smoothly more than once, the meridional cross sections

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