ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 4, Winter 1994

MIXED SIEGEL MODULAR FORMS AND KUGA FIBER VARIETIES

MIN HO LEE

1. Introduction

Mixed automorphic forms were first introduced by Hunt and Meyer [1] in connection with holomorphic forms on elliptic surfaces. A generalization to mixed automorphic forms of higher weights was treated in [5] (see also [7], [8]).

Let E be an elliptic surface and let $\pi\colon E \to X$ be an elliptic fibration in the sense of Kodaira (cf. [2]). Thus E is a compact smooth surface over $\mathbf{C}$, X is a compact Riemann surface, and the generic fiber of π is an elliptic curve. We assume that π has a global section and that there are no exceptional curves of the first kind in the fibers of π. Let E_0 be the union of the regular fibers of π and let $X_0 = \pi(E_0)$. We identify the universal covering space of X_0 with the Poincaré upper half plane $\mathscr{H}$, and the fundamental group $\pi_1(X_0)$ with a subgroup Γ of $PSL(2, \mathbf{R})$. Thus we have $X_0 = \Gamma \backslash \mathscr{H}$, where Γ acts on $\mathscr{H}$ by linear fractional transformations. Given a point $z \in X_0$, we choose a holomorphic 1-form on $E_z = \pi^{-1}(z)$ and a basis $\{\alpha_z, \beta_z\}$ of $H_1(E_z, \mathbf{Z})$ that depends on $z \in X_0$ in a continuous manner. Then the many-valued function

$$\omega(z) = \frac{\int_{\alpha_z} \Phi}{\int_{\beta_z} \Phi}$$

on X_0 can be lifted to a holomorphic function $\omega\colon \mathscr{H} \to \mathscr{H}$ satisfying $\omega(\gamma z) = \chi(\gamma)\omega(z)$ for all $\gamma \in \Gamma$ and $z \in \mathscr{H}$, where $\chi\colon \Gamma \to SL(2, \mathbf{R})$ is the monodromy representation of $\Gamma = \pi_1(X_0)$ for the elliptic fibration $\pi\colon E \to X$. Hunt and Meyer [1] defined the space of mixed cusp forms $S_{2,1}(\Gamma, \omega, \chi)$ using the automorphy factor

$$j(\gamma, z) = (cz + d)^2\big(c_\chi \omega(z) + d_\chi\big),$$

Received August 27, 1992.
1991 Mathematics Subject Classification. Primary 11F46; Secondary 14K10.