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THE SPLITTING THEOREM FOR ORBIFOLDS

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Introduction

In this paper we wish to examine a generalization of the splitting theorem of Cheeger-Gromoll [CG] to Riemannian orbifolds. Roughly speaking, a Riemannian orbifold is a metric space locally modelled on quotients of Riemannian manifolds by finite groups of isometries. The term *orbifold* was coined by W. Thurston [T] sometime around the year 1976–77. The term is meant to suggest the orbit space of a group action on a manifold. A similar concept was introduced by I. Satake in 1956, where he used the term *V-manifold* (see [S1]). The "V" was meant to suggest a cone-like singularity. Since then, orbifold has become the preferred terminology.

Recall that if M is a complete connected n-dimensional Riemannian manifold with nonnegative Ricci curvature that contains a line, then the Cheeger-Gromoll splitting theorem [CG] states that that M is isometric to $N \times \mathbf{R}$. Recall that a line is a unit speed geodesic $\gamma\colon \mathbf{R} \to M$ such that for any $s, t \in \mathbf{R}$, $d(\gamma(s), \gamma(t)) = |s - t|$.

THEOREM 1. *Let O be a complete n-dimensional Riemannian orbifold with nonnegative Ricci curvature. If O contains a line, then O splits isometrically as $O = N \times \mathbf{R}$ where N is a complete Riemannian orbifold with nonnegative Ricci curvature.*

THEOREM 2. *Let O be a compact Riemannian orbifold with nonnegative Ricci curvature and let $\tilde{O}$ denote its universal orbifold cover. Then $\tilde{O} = N \times \mathbf{R}^l$, where N is compact and $l \geq 0$. Also, there exists a short exact sequence*

$$1 \to F \to \pi_1^{\text{orb}}(O) \to C \to 1$$

where F is a finite group and C is a discrete cocompact group of isometries acting on $\mathbf{R}^l$. That is, C is a crystallographic group.

To prove these results we will need several results about orbifolds. All of these results can be found in the first author's Ph.D. thesis [B1]. A basic reference on general orbifolds is [T].

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