ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 4, Winter 1994

SPECTRAL ASYMPTOTICS OF FOLIATED MANIFOLDS

JAMES L. HEITSCH[1] AND CONNOR LAZAROV[2]

Introduction

In this paper we study the spectral distribution functions for the analytic and combinatorial leafwise Laplacians on a foliated manifold admitting a transverse invariant measure. We show (Theorem 4.1) that the dilational equivalence classes near zero of the two spectral distribution functions are the same. An immediate consequence is that this dilational equivalence class is independent of the metric (in the analytic case) and the bounded triangulation (in the combinatorial case) used to define it. Our main result (Theorem 2.6) is that this dilational equivalence class is invariant under a measure preserving leafwise homotopy equivalence. This result is equivalent to the invariance of the dilational equivalence class near infinity of the trace of the leafwise heat kernels.

Our results are motivated by those of [Ef-Sh], [G-Sh] and [Ef] concerning the equivariant homotopy invariance of the asymptotic behaviour of the spectral distribution function for Riemannian manifolds with a free isometric action of a discrete group. We extend many of the ideas of these papers to our situation. The main techniques used in the proof of the homotopy invariance, however, are those developed in [H-L2] (where we prove the leafwise homotopy invariance of the foliation betti numbers), the simplicial techniques of [D] and [W], and those of [H-L1].

It is a pleasure to thank J. Dodziuk for several helpful conversations.

1. Introductory material: von Neumann algebras of foliations, leafwise operators, triangulations and simplicial theory

Let F be a smooth oriented foliation of a compact oriented manifold M. We first recall briefly some facts about transverse measures. For details see [C], [M-S], chapter IV and [H-L1], §2.3. A transversal to F is a Borel subset of M which intersects each leaf in a countable set. A smooth transversal is a proper embedded submanifold of M which is also a transversal. The set of

Received August 17, 1992.
1991 Mathematics Subject Classification. Primary 57R30; Secondary 58C40, 58G25.
[1]Partially supported by a grant from the National Science Foundation.
[2]Partly supported by a PSC-CUNY grant.