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WEAK CONCENTRATION POINTS FOR MÖBIUS GROUPS

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0. Introduction

Although the limit sets of discrete groups of Möbius transformations have been studied for many decades, there are some very natural dynamical properties of limit points which have not been previously examined. In this paper, we use geometric and topological methods to study one of these, which we call the property of being a weak concentration point. Stronger concentration conditions are treated in [A-H-M] and [M1]. In [A-H-M], the limit points satisfying a certain strong concentration condition are characterized in several ways, each analogous to a classical characterization of conical limit points. This condition is strictly stronger than being a conical limit point, although for a large class of groups the points satisfying it have full measure in the conical limit set. In [M1], an intermediate concentration condition is found which is exactly equivalent, for finitely generated Fuchsian groups, to the property of being a conical limit point.

Weak concentration is the simplest and perhaps most natural of the concentration conditions. To fix notation, let Γ be a nonelementary discrete group of hyperbolic isometries acting on the Poincaré disc D^m, $m \geq 2$, and let $p \in \partial D^m$ be a limit point of Γ. By a neighborhood of p, we will always mean an open neighborhood of p in ∂D^m.

DEFINITION. An open set U in ∂D^m *can be concentrated* at p if for every neighborhood V of p, there exists an element $\gamma \in \Gamma$ such that $p \in \gamma(U)$ and $\gamma(U) \subseteq V$.

Equivalently, U can be concentrated at p if and only if the set of translates of U contains a local basis for the topology of ∂D^m at p. As we show in Theorem 1.1, every limit point of Γ has a disconnected neighborhood which can be concentrated. Therefore we restrict attention to concentration of connected neighborhoods.

DEFINITION. The limit point p is called a *weak concentration point* for Γ if there exists a connected open set U that can be concentrated at p.

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