ORIENTATION REVERSING AUTOMORPHISMS OF RIEMANN SURFACES

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It was shown by Jakob Nielsen [N] that the fixed point data determines an orientation preserving automorphism of prime order on a given compact Riemann surface up to topological conjugacy. In this paper we classify up to topological conjugation the orientation reversing automorphisms of order $2p$, for p prime, on compact Riemann surfaces of genus $g_0 \geq 2$. In 1979, Robert Zarrow studied this classification (see [Z1] and [Z2]). However we have found some errors in his works.

We separate our study in two cases: when the automorphisms have order 4 and when the automorphisms have order $2p$, with p an odd prime. In the first case we have proved the following theorem:

THEOREM 1. *Let X be a Riemann surface, suppose that ϕ_1 and ϕ_2 are two orientation reversing automorphisms of X such that ϕ_1^2 and ϕ_2^2 have order 2 and they have fixed points. Then ϕ_1 and ϕ_2 are conjugate if and only if ϕ_1^2 and ϕ_2^2 have the same number of fixed points.*

The above theorem agrees with Theorem 1.1 of [Z1] but if the considered automorphisms have fixed point free squares and $g_0 \equiv 1 \pmod 4$ then we find two conjugacy classes instead of one as Zarrow claimed (see Theorem 2).

For the automorphisms of order $2p$ with p an odd prime we have established the following result:

THEOREM 3. *Let X be a Riemann surface and suppose that ϕ_1 and ϕ_2 are two orientation reversing automorphisms of order $2p$ where p is an odd prime. Then ϕ_1 and ϕ_2 are conjugate if and only if* (1) *$X/\langle\phi_1\rangle$ and $X/\langle\phi_2\rangle$ are homeomorphic,* (2) *ϕ_1^2 and ϕ_2^2 are conjugate and* (3) *the action of ϕ_1^2 on Fix ϕ_1^p (fixed point set of ϕ_1^p) is conjugate to the action of ϕ_2^2 on Fix ϕ_2^p.*

The conditions of this theorem are different to those proposed in [Z2]. However in the example in Section 3 we show that the conditions of Zarrow's statement are not sufficient.