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ON n-WIDTHS OF CLASSES OF HOLOMORPHIC FUNCTIONS WITH REPRODUCING KERNELS

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1. Introduction

Let A be a subset of a Banach space X. The Kolmogorov n-width of A in X is defined by

$$d_n(A, X) = \inf_{X_n} \sup_{x \in A} \inf_{y \in X_n} \|y - x\|$$

where X_n varies over all subspaces of X of dimension n.

The Gel'fand n-width is given by

$$d^n(A, X) = \inf_{Y_n} \sup_{x \in A \cap Y_n} \|x\|$$

where Y_n runs over all closed subspaces of X of codimension n.

The linear n-width is defined by

$$\delta_n(A, X) = \inf_{T_n} \sup_{x \in A} \|x - T_n x\|$$

where T_n runs over all linear operators of X into itself which have rank n or less. There are evident inequalities among these quantities; namely,

$$\delta_n(A, X) \geq d_n(A, X), \tag{1}$$

and

$$\delta_n(A, X) \geq d^n(A, X). \tag{2}$$

The concept of the n-width of a set was originally introduced by Kolmogorov [6] in 1936. Widths are important in approximation theory since knowledge of the exact or even the asymptotic value of the n-width can lead to best or near best methods of approximation and interpolation, as well as to the estimation of errors in these methods. Moreover, determination of an optimal subspace typically gives optimal methods of approximation, as well as fascinating

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