LIKELIHOOD RATIOS FOR STOCHASTIC PROCESSES RELATED BY GROUPS OF TRANSFORMATIONS II

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We will make use of the notation established in *Likelihood Ratios for Stochastic Processes Related by Groups of Transformations*[2] (referred to as (I) in the following). Thus, X, S, and P are a set, a σ-algebra of subsets, and a probability measure on S. T_α is a one-parameter group of automorphisms of an algebra F of bounded, real-valued, S-measurable functions satisfying

(i) T_α preserves bounds, and $T_\alpha f(x)$ has a continuous derivative $D(T_\alpha f)(x)$ in α which is bounded uniformly in α and x for every f in F and x in X, and

(ii) if f_n is a uniformly bounded sequence from F with $\lim f_n(x) = 0$ for all x, then $\lim T_\alpha f_n(x) = 0$ for all x.

Examples of this situation will be found in (I).

We will write P_α for the measures which are the completions of

$$l_\alpha(f) = \int T_\alpha f \, dP,$$

$K_\sigma(\alpha)$ for the Gaussian kernel $(2\pi\sigma)^{-1/2} \exp(-\alpha^2/2\sigma)$, and P_α^σ for the measures which are the completions of

$$l_\alpha^\sigma(f) = \int_{-\infty}^{\infty} K_\sigma(\beta) \left(\int T_{\alpha+\beta} f \, dP \right) d\beta.$$

According to Theorem 4.2 of (I) the P_α^σ with $\sigma > 0$ and any α are mutually absolutely continuous, and for each positive σ there is a ϕ^σ in $L_1(P^\sigma)$ satisfying

$$\int \phi^\sigma T_\alpha f \, dP^\sigma = \frac{\partial}{\partial \alpha} \int T_\alpha f \, dP^\sigma$$

for f in F and

$$\log \frac{dP_\alpha^\sigma}{dP^\sigma} = \int_0^\alpha T_{-\beta} \phi^\sigma \, d\beta.$$

The theorem also asserts that the transformations $V^\sigma(\alpha)$ on $L_1(P^\sigma)$ defined by the equation $V^\sigma(\alpha)f = (dP_\alpha^\sigma/dP^\sigma)T_{-\alpha} f$ for f in F form a strongly continuous one-parameter group of isometries whose infinitesimal generator A^σ is defined on F and satisfies $A^\sigma f = \phi^\sigma f - Df$ there.

We note that $\bar{F}$, the set of uniform limits from F, contains the functions $f \wedge g = \min(f, g)$ and $f \vee g = \max(f, g)$ whenever it contains f and g,

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[2] Illinois Journal of Mathematics, vol. 7 (1963), pp. 396–414.