GROUPS WHOSE IRREDUCIBLE REPRESENTATIONS HAVE DEGREES DIVIDING p^e

BY

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Let G be a finitely generated group and $C[G]$ its group algebra over the complex numbers C. In this paper we consider groups with the property that the degrees of all the irreducible representations of $C[G]$ divide a fixed prime p to the power e. This is a special case of the situation studied in [4]. In fact, our result, Theorem I, is a sharper version of Theorem III of that paper. In the more special case $e = 1$, Theorem II gives necessary and sufficient conditions on the structure of the group. For $p = 2$ this yields in particular Theorem 3 of [1].

Our main results are:

Theorem I. *Let G be a finitely generated group and p a prime. Suppose that all irreducible representations of G over the complex numbers have degrees dividing p^e. Then G has a subinvariant series*

$$G = A_e \supseteq A_{e-1} \supseteq \cdots \supseteq A_0$$

such that A_0 is abelian and A_i/A_{i-1} is elementary abelian p with not more than $2i + 1$ generators. Hence G has an abelian subgroup A_0 whose index divides $p^{e(e+2)}$.

Theorem II. *Let G be a finitely generated group all of whose irreducible representations have degree 1 or p. Then G is one of the following types:*

1. *G is abelian.*
2. *G has a normal abelian subgroup of index p.*
3. *G has a center Z with G/Z being a group of order p^3 and period p.*

Conversely, let G be one of the above. If G is finite then all of its irreducible representations have degree 1 or p. If G is finitely generated then G at least has a complete set of representations of degree 1 or p.

In Section 4 we give examples to show that all of the above types can occur.

1. In this section we fix nomenclature and give some character-theoretic propositions which are basic to the rest of the paper. All groups in this paper are assumed to be finite unless otherwise stated.

Let χ be an irreducible character of a group G and φ an irreducible character of a subgroup H of G. φ induces a character φ^* of G and χ restricts to a character $\chi \mid H$ of H. From the Frobenius Reciprocity Theorem [3, Theorem 38.8] we can conclude that the multiplicity of χ as a constituent of φ^* is equal to the multiplicity of φ as a constituent of $\chi \mid H$.

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